Workshop on Modern Applied Mathematics PK 2013

Kraków 18 — 20 October 2013

Warsztaty z Nowoczesnej Matematyki i jej Zastosowań

Kraków 18 — 20 października 2013

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Contents

Introduction

Workshop on Modern Applied Mathematics PK 2013 is an annual conference on modern mathematics organized by the Institute of Mathematics of the Faculty of Physics, Mathematics and Computer Science, Tadeusz Kościuszko Cracow University of Technology.

The Conference aims to present new results, to promote and to bring together researchers in the different research areas of mathematics and influence more cooperation among scientists working in mathematics. This conference will provide a unique forum for exchanging ideas and in-depth discussions on different aspects and different branches of mathematics.

You can find detailed information about conference on the page:

`www.wmam.pk.edu.pl`

After the conference a special issue of Technical Transactions with will be published and every participant is invited to submit an article.

I would like to thank all participants for interest in our conference and scientific research in the field of mathematics.

I would like to express my thanks to Board of Directors and the Administration of the Institute of Mathematics as well as the staff of the Institute for their friendliness and support for conference organization.

Grzegorz Gancarzewicz

Warsztaty z Nowoczesnej Matematyki i jej Zastosowań są coroczną konferencją organizowaną przez Instytut Matematyki na Wydziale Fizyki, Matematyki i Informatyki Politechniki Krakowskiej im. Tadeusza Kościuszki. Celem konferencji jest prezentacja aktualnych osiągnięć naukowych w zakresie matematyki i jej zastosowań, promocja matematyki i badań naukowych w tej dziedzinie, spotkanie i wymiana doświadczeń przez naukowców zajmujących się różnymi działami matematyki.

Szczegółowe informacje o konferencji znajdują się na stronie:

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Po konferencji zostanie wydany specjalny numer czasopisma Technical Transactions. Zapraszamy wszystkich uczestników konferencji do złożenia artykułów do publikacji.

Bardzo dziękuję wszystkim uczestnikom za zainteresowanie konferencją i badaniami naukowymi w dziedzinie matematyki.

Chciałbym wyrazić szczególne podziękowania Dyrekcji Instytutu Matematyki, administracji Instytutu Matematyki i wszystkim pracownikom za życzliwość i pomoc w organizacji konferencji.

Grzegorz Gancarzewicz

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Abstracts

Unique dot pattern

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Zagadnienie unikalnego rozkładu kropek to ciekawy problem kombinatoryczny mający zastosowanie przy użyciu długopisów cyfrowych. Chodzi o to, jak zakodować powierzchnię przy pomocy miniaturowych kropek w taki sposób, aby kodowanie było jednoznaczne, istniała funkcja dekodująca, a zakodowana powierzchnia była jak największa. Kropki mogą być ustawiane w jednej z czterech pozycji na predefiniowanej siatce, a kodowanie jednej współrzędnej składa się z kwadratu kropek o rozmiarze 10x10. Podczas referatu przedstawiona zostanie formalna definicja problemu, jego częściowe rozwiązanie oraz postawiony zostanie problem otwarty związany z ograniczeniem górnym na wielkość możliwej do uzyskania powierzchni.

On some theorem of Professor Teresa Winiarska

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Celem referatu jest przedstawienie twierdzenia Pani Profesor Teresy Winiarskiej z monografii [1] na temat istnienia i jednoznaczności klasycznego rozwiązania abstrakcyjnego zagadnienia ewolucyjnego

$$u'(t) + Au(t) = k(t),\ t \in (t_0, t_0 + a],$$

$$u(t_0) = x,$$

gdzie $-A$ jest infinitezymalnym generatorem półgrupy klasy C_0 na przestrzeni Banacha.

W trakcie referatu zostanie pokazane zastosowanie twierdzenia Pani Profesor Teresy Winiarskiej do rozwiązywania semi-liniowych abstrakcyjnych zagadnień Cauchy'ego.

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On the Ficken criterion on norms induced by inner products

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We will discuss some criteria ensuring that a norm is induced by an inner product including the ones given by Ficken and Danelich. The talk is motivated mostly by the annoying incomplete proof of one of these conditions circulating in the mathematical literature.

Complex multiplication

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As topological spaces or even real manifolds all elliptic curves are the same, they are diffeomerphic to a standard torus $C^1 \times C^1$. As complex manifolds they form a one parameter family parametrised by the so–called j–invariant. For two particular values of j–invariant 0 and $1728 = 2^6 3^3 = 12^3$ we get very special curves with Weierstrass equations

$$y^2 = x^3 - 1 \qquad \text{and} \qquad y^2 = x^3 - x$$

These two curves quotients of the complex plane by two most symmetric lattices: tessellation into squares and regular triangles. The remarkable property of both curves is the existence of special symmetries, expect the usual symmetry of Weierstrass equation

$$(x, y) \mapsto (x, -y)$$

they admit respectively

$$(x, y) \mapsto (e^{\frac{2\pi i}{3}} x, y) \qquad \text{and} \qquad (x, y) \mapsto (-x, iy).$$

Surprisingly, there exist infinite (countable) many elliptic curves which admit slightly weaker property, the so–called complex multiplication. They turned out to be easier for most of the classical problems concerning elliptic curves, I will shortly discuss these curves, pointing out their relations with unique factorization imaginary quadratic fields and especially the beautiful almost integer

$$e^{\pi\sqrt{163}} \approx 262537412640768743,99999999999992500725.$$

Finally, I will explain application of elliptic curves with complex multiplication to construction of some special Calabi–Yau threefolds (joint work [1] with Matthias Schütt, Leibniz Uniwersität Hannover, Germany).

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Statistical problems for periodic Orsntein Uhlenbeck process

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In this work we investigate estimation problems for a diffusion process following the model

$$d\xi_t = f(t)\xi_t\,dt + d\mathrm{B}_t$$

where $f : \mathbf{R} \to \mathbf{R}$ is a periodic continuous function with period $P > 0$, when the process is observed through continuous time $[0, T]$ as $T \to \infty$.

First consider that the drift function $f(\cdot)$ depends linearly on an unknown parameter $\theta \in \mathbf{R}$: $f(t) = \theta b(t)$, $b : \mathbf{R} \to \mathbf{R}$ being a known periodic continuous function. Then the maximum likelihood estimator (MLE) of θ is consistent and we point out its asymptotic minimax efficiency. These results comply with the well-established case when the function $f(\cdot)$ is constant non null. However the case when $\int_0^P b(t)dt = 0$ and $b(\cdot)$ is not identically null presents some particularities. For instance in this case whatever is the value of θ, the rate of convergence of the MLE is T as in the case when $\theta = 0$ and $\int_0^P b(t)dt \neq 0$. Futhermore when $\int_0^P b(t)dt = 0$, the MLE is locally efficient for the quadratic risk.

Next we deal with the problem of nonparametric estimation of the drift function $f(\cdot)$, its period P being known. We construct a kernel estimator of $f(\cdot)$ which is consistent. Its rate of convergence depends on the signum of $\int_0^P f(t)dt$ as in the previous parametric problem. (joint work with Khalil El Waled).

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Subsampling method for weakly dependent and periodically correlated sequences

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The weak dependence - a type of dependence in time series - introduced by P. Doukhan in 1999 gives the tools for the analysis of statistical procedures with very general data generating processes. One of such statistical procedures is subsampling.

Subsampling can be used if statistical inference for dependent data based on asymptotic distributions fails or there are problems with sample size. To apply subsampling it is sufficient to know if there exists a non-degenerated asymptotic distribution of the statistic.

For independent data and stationary time series subsampling procedures are well investigated. Our research is focused on non-stationary periodically correlated time series.

In the presentation the generalization of the model introduced by Politis and McElroy in 2007 is considered for periodically correlated processes with a known period.

The model investigated in the presentation is: $X_t = \sigma_t G_t + \eta_t$, where σ_t and G_t are independent, σ_t, is i.i.d. mean μ different from zero and has the marginal distribution of an $\alpha-$stable random variable. Moreover G_t is periodically correlated time series with known period T and can be written as $G_t = f_t N_t$ for a long memory, stationary mean zero Gaussian process N_t, and f_t- bounded and scalar periodic sequence $f_t = f_{t+T}$, $\eta_t (= \eta(t))$ is periodic with the same, known period T as f_t.

In such model, the joint asymptotic behavior of the sample mean and the sample variance is investigated. The weak dependence property gives the tools to improve the subsampling consistency of self-normalized statistics.

Additionally the simulations will be given.

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On the value sharing of rational functions and Erdős-Woods Conjecture

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Mówimy, że dwie nie stałe funkcje meromorficzne f, g współdzielą pewną wartość $a \in \hat{\mathbb{C}}$, gdy mają to samo włókno nad a tzn. $f^{-1}(\{a\}) = g^{-1}(\{a\})$. Nevallina [3] udowodnił, że dwie nie stałe funkcje meromorficzne $f, g : \mathbb{C} \to \hat{\mathbb{C}}$, które współdzielą pięć różnych wartości muszą być równe. W referacie skupimy się na współdzieleniu wartości funkcji wymiernych $f, g : \hat{\mathbb{C}} \to \hat{\mathbb{C}}$. Przedstawimy dowód Sauera [4], że funkcje wymierne, które współdzielą cztery wartości muszą być równe. Istnieją jednak pary różnych funkcji wymiernych, które współdzielą trzy wartości. Przedstawimy numeryczne wyniki dotyczące takich par [1]. Wskażemy analogię między twierdzeniami o współdzieleniu wartości funkcji a hipotezą Erdősa-Woodsa: Istnieje taka stała $k > 2$, że dla dowolnych liczb naturalnych x, y równości $\mathrm{rad}(x + i) = \mathrm{rad}(y + i)$ dla $i = 1, 2, \ldots, k$ implikują $x = y$ (gdzie $\mathrm{rad}(n)$ oznacza iloczyn liczb pierwszych dzielących n). Przedstawimy prosty dowód hipotezy Erdősa-Woodsa, przy założeniu hipotezy ABC. Pierwszy dowód tej implikacji pochodzi od Langevina [2].

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Regression trees for longitudinal data

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In longitudinal studies, repeated measurements of the outcome variable are often collected at irregular and possibly subject-specific time points. Parametric regression methods for analyzing such data have been developed by Laird and Ware (1982) and Liang and Zeger (1986) among others. Often the population under consideration is heterogeneous in terms of the longitudinal trends and their dependence of the baseline covariates. In such situations traditional mixed effect models, such as linear mixed effects models, might not capture the complex longitudinal trend interactions. If the population under consideration is diverse and there exist several distinct subgroups within it, the true parameter value(s) quantifying the longitudinal changes may vary between these subgroups. This is often the case in observational studies with many possible predictors. In such cases, a group-averaged trajectory will mask important subgroup differences. Our aim is to identify and characterize longitudinally homogeneous subgroups based on the combination of baseline covariates (see Kundu and Harezlak, 2013). We achieve this goal by constructing regression trees through binary partitioning, choosing the best split by repetitive evaluation of a goodness of fit criterion at all splits of partitioning variables. To remedy the problem of multiple testing for each split, we perform a single test to identify the instability of parameter(s) in longitudinal models. We obtain asymptotic results and examine finite sample behavior of our method through simulation studies. Finally, we apply our method to study the changes in brain metabolite levels of HIV infected patients.

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The existence of a weak solution of the semilinear second-order differential equation in a Banach space

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We consider the abstract second-order initial value problem

$$\frac{d^2}{dt^2}u(t) = Au(t) + f(t, u(t)) \quad \text{for} \quad t \in (0, T], \tag{1}$$

$$u(0) = x, \tag{2}$$

$$u'(0) = y. \tag{3}$$

where A is a densely defined, closed linear operator on a Banach space X, $x, y \in X$ and $f : [0, T] \times X \to X$. For a Banach space X, X^* will denote its dual space. Let $\langle \cdot, \cdot \rangle : X \times X^* \longrightarrow \mathbb{K}$ be the duality pairing. For an operator A, $D(A)$ and A^* will denote its domain and adjoint, respectively.

Definition 1. A function $u \in C([0, T];\ X)$ is a weak solution of the problem (1)–(3) on $[0, T]$ if for every $v \in D(A^*)$ the functions $[0, T] \ni t \to \langle u(t), v\rangle$, $[0, T] \ni t \to \frac{d}{dt}\langle u(t), v\rangle$ are absolutely continuous on $[0, T]$ and

$$\frac{d^2}{dt^2}\langle u(t), v\rangle = \langle u(t), A^*v\rangle + \langle f(t, u(t)), v\rangle \ a.e. \text{ on } [0, T],$$

$$u(0) = x,$$

$$\frac{d}{dt}\langle u(t), v\rangle \mid_{t=0} = \langle y, v\rangle.$$

Definition 2. Let A be a generator of a strongly continuous cosine family $\{C(t)\}_{t\in\mathbb{R}}$. A function $u \in C([0, T];\ X)$ is a mild solution of the problem (1)–(3) on $[0, T]$ if it is a solution of the integral equation

$$u(t) = C(t)x + S(t)y + \int_0^t S(t - s)f(s, u(s))ds,$$

where $\{S(t)\}_{t\in\mathbb{R}}$ is the sine family associated with $\{C(t)\}_{t\in\mathbb{R}}$.

We are concerned with two types of solutions: weak and mild. Under the assumption that A is the generator of a strongly continuous cosine family, we establish sufficient conditions such that if u is a weak (mild) solution of that initial value problem, then u is a mild (weak) solution of that problem.

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Optimizing the employment of production workers in manufacturing seasonal conditions and limitations of the Labour Code

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Podczas referatu zostanie zaprezentowany model matematyczny problemu szeregowania zadań związany z optymalizacją poziomu zatrudnienia pracowników bezpośrednio produkcyjnych w warunkach różnej sezonowości oraz ograniczeń wynikających z kodeksu pracy obowiązującego w Polsce na sierpień 2013 r. Funkcją celu jest funkcja minimalizująca koszt przy spełnieniu licznych założeń związanych z prawem pracy oraz koniecznością zaspokojenia zapotrzebowania na dany produkt. Funkcja ta, w wersji uproszczonej przedstawia się następująco:

$$\sum_{m\in M}\sum_{o\in O} s(czt,o,m)\cdot\left[\sum_{p\in P} f(p,\bigcup_{j\in o} zm2(m,j))\cdot k(p,czt)\cdot 8\cdot \max_{l\in\bigcup_{j\in o} zm2(m,j)} m_1(m,l)\right]$$

$$+\sum_{m\in M}\sum_{o\in O} s(zm,o,m)\cdot\left[\sum_{d\in D_1\setminus\{\text{czterobrygadówka}\}}\sum_{p\in P} f(p,\bigcup_{j\in o} zm1(m,j,d))\cdot k(p,d)\cdot 8\cdot\right.$$

$$\left.\max_{l\in\bigcup_{j\in o} zm1(m,j,d)} m_2(m,l)+\sum_{d'\in D_2}\sum_{i\in\mathcal{P}(\bigcap_{k\in o} ol(k))}\sum_{p\in P} f(p,i)\cdot k(p,d')\cdot nadg(d',i,m)\right]$$

a jej wytłumaczenie znajdzie miejsce podczas referatu. Opracowany model oraz funkcja zostały zastosowane w jednej z dużych poznańskich firm produkcyjnych.

Fattening effect in $\mathbb{P}^1 \times \mathbb{P}^1$- part I: α^+ invariant

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Bocci and Chiantini in [1] studied the effect of points fattening for space $\mathbb{P}^2$. The projective plane is an instance of a del Pezzo surface. Another simple del Pezzo surface is the product of two projective lines $\mathbb{P}^1 \times \mathbb{P}^1$. Therefore it seems natural to consider the issue of initiated by them "*α–problem*" in $\mathbb{P}^1 \times \mathbb{P}^1$.

The *initial degree* $\alpha(Z)$ of a set of points $Z \subset \mathbb{P}^2$ is defined as the least degree of a curve passing through all points of Z. In general $\alpha(mZ)$ for some positive $m \in \mathbb{Z}$ is the least degree of a curve passing through all points of Z with multiplicity m (in other words *vanishing along* mZ).

By an analogy to this definition we could intuitively say, that the *initial degree* of a set of points in $\mathbb{P}^1 \times \mathbb{P}^1$ is the least bi–degree of a curve vanishing along this set. But here a question arises — what does it exactly mean for bi–degree to be the smallest?

There are two possible natural ways of definition of the *initial degree* $\alpha(Z)$ in the space $\mathbb{P}^1 \times \mathbb{P}^1$. One can see both of these definitions in [2]. In this part we are presenting results related to one of this definition, denoted by $\alpha^+(Z)$.

Definition 1. Let $Z \subset \mathbb{P}^1 \times \mathbb{P}^1$ be a set of points. We define

$$\alpha^+(mZ) = \min\{k = k_1 + k_2 : \text{exists a curve } C \text{ of bi–degree } (k_1, k_2) \text{ vanishing along } mZ\}.$$

This version of the *initial degree* in the space $\mathbb{P}^1 \times \mathbb{P}^1$ has similar properties and behaves as the function α on projective plane $\mathbb{P}^2$.

We present here main results we obtained for *initial degree* in $\alpha^+(Z)$ version.

Theorem 2. Let $Z \subset \mathbb{P}^1 \times \mathbb{P}^1$ be a set of points. The function α is strictly increasing as a function of m, i.e. the following strong inequality holds

$$\alpha^+(mZ) < \alpha^+((m+1)Z).$$

This theorem shows, that there is not possible to obtain a jump of 0 between two consecutive α^+'s, for any configuration of points $Z \subset \mathbb{P}^1 \times \mathbb{P}^1$.

The following theorem gives a characterization of configurations of points with the minimal possible jump between next α^+'s, namely a jump by 1.

Theorem 3. Let $Z \subset \mathbb{P}^1 \times \mathbb{P}^1$ be a set of points. Assume that

$$\alpha^+((m+1)Z) = \alpha^+(mZ) + 1$$

for some integer $m \geq 1$. Then all points of Z lie on a single vertical or horizontal fiber.

Detailed proofs of these results are presented in [2].

Definition and results related to second variant of definition of *initial degree* in space $\mathbb{P}^1 \times \mathbb{P}^1$ will be presented in lecture *Fattening effect in* $\mathbb{P}^1 \times \mathbb{P}^1$*- part II:* α^* *invariant.*

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Nonstochastic modelling for time series and signals

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The aim of this talk is to introduce relative measures as tools in modelling signals, time series and random processes. The research related to almost periodic functions and their generalizations leads in a natural way to the definition of the alternative probability space that is generated by single realizations of observed signals, time series and processes. The natural concepts like: distribution function, expectation, higher order moments have their natural equivalents in the probability space generated by relative measure. Moreover, the recent research of Dehay, Napolitano and Leskow has proved an equivalent central limit theorem.

Fattening effect in $\mathbb{P}^1 \times \mathbb{P}^1$- part II: α^* invariant

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In part I, we saw some results for α^+. Using previous notations in this part we consider α^* as follows

Definition 1. For a set of points $Z \subset \mathbb{P}^1 \times \mathbb{P}^1$ we define

$$\alpha^*(mZ) = \min\{k : \text{there exists a curve } C \text{ of bi–degree } (k,k) \text{ vanishing along } mZ\}.$$

In contrast to the α^+ where always $\alpha^+(mZ) < \alpha^+((m+1)Z)$ holds, it might happen that $\alpha^*((m+1)Z) = \alpha^*(mZ)$.

There is a strong geometrical constrain under which this equality is possible.

Theorem 2. Let $Z = \{P_1, ..., P_s\}$. The following conditions are equivalent

a) $\alpha^*((m+1)Z) = \alpha^*(mZ)$ for some $m \geq 1$;

b) there exist finite sets $Z_V, Z_H \subset \mathbb{P}^1$ such that $Z = Z_V \times Z_H$, i.e. Z is a *grid.*

The immediate consequence of the theorem is the following corollary

Corollary 3. There is no set of points Z such that the equality

$$\alpha^*((m+2)Z) = \alpha^*((m+1)Z) = \alpha^*(mZ)$$

holds for any positive integer m.

It is interesting to characterize also sets of points in $\mathbb{P}^1 \times \mathbb{P}^1$ with the next minimal jump of the α^*- value. I.e. sets with $\alpha^*((m+1)Z) = \alpha^*(mZ) + 1$. This is a difficult problem in general, however using results of Chudnovsky relating Waldschmidt constants which we define for homogeneous ideals $I \subset \mathbb{C}[\mathbb{P}^n]$ of points Z as an asymptotic counterpart of the initial degree

$$\gamma(I) = \lim_{m\to\infty} \frac{\alpha^*(mZ)}{m},$$

we may show that

$$\frac{\alpha^*(mZ)}{m} \geq \frac{\alpha^*(Z)}{2},$$

and in the consequence we claim that

Theorem 4. Let $Z = \{P_1, \dots, P_s\}$be the set of points. Assume that

$$\alpha^*(6Z) = \alpha^*(Z) + 5.$$

Then $\alpha^*(Z) = 1$.
Moreover the sequence of equalities cannot be shortened in general.

All proofs in details of these results can be found in [2].

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Practical aspects of data correlation in business fraud detection

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Events processing from heterogeneous environments is in practical applications difficult to implement because of limited on-line processing and reasoning capabilities caused by a huge amount of information (hundreds GB per h of data). In [2] we defined a few stages of efficient data processing. It starts from different sources and source types, from where the data are gathered using preprocessing rules. This leads to a general fact form stored in a repository of facts. This general form allows us to use independent processing methods in all aggregation, enrichment and correlation steps. Event extraction from the repository of facts is based on rules defined in the processing engine. The extracted events create streams where the events will be subject to future correlation and aggregation steps. Separation of events from facts and forcing to create the event streams will significantly reduce the computational complexity of both operations. The key processing element is the correlation engine based on the Rete or the Rete-OO algorithm [2]. The usage of Rete or Rete-OO algorithm is motivated by the possibility of correlation coincidence and the optimization of aggregation conditions tree. The engine allows us to define correlation and aggregation conditions based on CEP (Complex Event Processing) rules [1].

In the talk both EDA (Event Driven Architecture) and SOA (Service Oriented Architecture) architecture models will be shortly described, as parental ones to our solution. We will also present some commercial methods of data analysis and business processing in the context of the business fraud detection. The data analysis is based on events collected from multiple applications in a distributed production environment. The system architecture will be described and compared to an exemplary embodiment of engines for rule-based event processing.

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Air pollution model

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Podczas referatu przedstawiony zostanie matematyczno-fizyczny model rozprzestrzeniania się lotnych substancji zagrażających zdrowiu i życiu ludzi. W skutek przeprowadzonych badań, który z istniejących modeli matematycznych będzie najlepiej odpowiadał podanym w celu wymaganiom, przyjęto gaussowski model rozprzestrzeniania się chmury. Istnieją dwa modele gaussowskie: model emisji ciągłej i model emisji chwilowej. Emisja chwilowa odnosi się do krótkotrwałego wypływu gazu, przy której zakładamy zerowy czasu uwolnienia wybuchu. Jest to na przykład wybuch, w którym pojawia się chmura lub nagłe rozszczelnienie zbiornika. Emisja ciągła odnosi się sytuacji, w której gaz emitowany jest ze stałą prędkością przez dłuższy okres czasu, jest to np. gaz wydobywający się z komina lub rozszczelniona rura, z której ciągle wydobywa się gaz. Dla emisji ciągłej przyjęty jest wzór, który służy do obliczenia stężenia w określonym punkcie (x, y, z):

$$C = \frac{G}{2\pi\sigma_y\sigma_z u}\left[\exp\frac{-y^2}{2\sigma_y^2}\right]\left[\exp\frac{-(z-H)^2}{2\sigma_z^2} + \exp\frac{-(z+H)^2}{2\sigma_z^2}\right]$$

Dla emisji chwilowej przyjęty jest wzór, który służy do obliczenia stężenia w określonym punkcie (x, y, z):

$$C = \frac{M}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z}\left[\exp\left(-\frac{(x-ut)^2}{2\sigma_x^2} - \frac{-y^2}{2\sigma_y^2}\right)\right]\left[\exp\frac{-(z-H)^2}{2\sigma_z^2} + \exp\frac{-(z+H)^2}{2\sigma_z^2}\right]$$

Ponadto dla emisji ciągłej pod uwagę bierze się tylko te punkty, które spełniają warunek:

$$\frac{y^2 + (z-H)^2}{x^2} << 1$$

Podczas referatu zostaną przedstawione szczegółowe modele.

Doppler Effect on Almost-Cyclostationary Signals

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Almost all modulated signals adopted in communications, radar, sonar, and telemetry can be modeled as *almost cyclostationary (ACS)*. That is, the (conjugate) autocorrelation function of their complex envelope $x(t)$ is an almost-periodic function of time

$$\mathrm{E}\left\{x(t+\tau)\, x^{(*)}(t)\right\} = \sum_{\alpha \in A} R^{\alpha}_{xx^{(*)}}(\tau)\, e^{j2\pi\alpha t} \tag{4}$$

where $(*)$ denotes an optional complex conjugation, A is the countable set (depending on $(*)$) of possibly incommensurate (conjugate) cycle frequencies, and the Fourier coefficients

$$R^{\alpha}_{xx^{(*)}}(\tau) \triangleq \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathrm{E}\left\{x(t+\tau)\, x^{(*)}(t)\right\}\, e^{-j2\pi\alpha t} \mathrm{d}t \tag{5}$$

are referred to as *(conjugate) cyclic autocorrelation functions* [1]. Equivalently, ACS signals have spectral components separated by $\alpha \in A$ that are correlated. That is, the *Loéve bifrequency spectrum*

$$\mathrm{E}\left\{X(f_1)\, X^{(*)}(f_2)\right\} = \sum_{\alpha \in A} S^{\alpha}_{xx^{(*)}}(f_1)\, \delta\Big(f_2 - {\scriptstyle(-)}(\alpha - f_1)\Big) \tag{6}$$

where $\delta(\cdot)$ is Dirac delta and ${\scriptstyle(-)}$ denotes an optional minus sign linked to $(*)$, has support contained in lines with slope ± 1. In (6), $X(f)$ denotes the Fourier transform of the signal $x(t)$ defined in a generalized sense [2, Sec. 1.1.2, 4.2.1] and the functions $S^{\alpha}_{xx^{(*)}}(f_1)$, referred to as *(conjugate) cyclic spectra*, are the densities of spectral correlation along the support lines $f_2 = {\scriptstyle(-)}(\alpha - f_1)$, $\alpha \in A$. The (conjugate) cyclic spectra are the Fourier transforms of the (conjugate) cyclic autocorrelation functions.

In the case of free-space propagation and wide-band transmitter and receiver antennas, the relative motion between transmitter and receiver induces on the transmitted signal a time-varying gain $A(t)$ and a time-varying delay $D(t)$ [2, Sec. 7.1.3]. Under mild conditions the time-varying gain can be considered constant and the complex envelope of the received signal is given by

$$y(t) = A\,x(t - D(t))\,e^{-j2\pi f_c D(t)} \tag{7}$$

where f_c is the carrier frequency.

The Doppler effect is due to the time-varying delay $D(t)$. It modifies the kind of nonstationarity of the transmitted signal and determines the joint nonstationary characterization of transmitted and received signals.

In the case of constant relative radial speed between transmitter and receiver, $D(t)$ is a linear function of t and the signal $y(t)$ can be written as

$$y(t) = b\,x(st - d)\,e^{j2\pi\nu t} \tag{8}$$

where b is a complex gain, s a time-scale factor, d a delay, and ν a frequency shift. Its Loéve bifrequency spectrum is given by

$$\begin{aligned} \mathrm{E}\left\{Y(f_1)\,Y^{(*)}(f_2)\right\} &= \frac{bb^{(*)}}{|s|}\,e^{-j2\pi(f_1-\nu)d/s}\,e^{-(-)j2\pi(f_2-\nu)d/s} \\ &\quad \sum_{\alpha\in A} S^{\alpha}_{xx^{(*)}}\left(\frac{f_1-\nu}{s}\right)\delta\Big(f_2 +_{(-)} f_1 - \left[{}_{(-)}s\alpha + \kappa\nu\right]\Big) \end{aligned} \tag{9}$$

where $\kappa \triangleq (1 +_{(-)} 1)$. Thus, the received signal is ACS. In contrast, the transmitted and received signals are *jointly spectrally correlated (SC)* [2, Chap. 4]. That is, their Loéve bifrequency cross-spectrum is

$$\mathrm{E}\left\{Y(f_1)\,X^{(*)}(f_2)\right\} = \frac{b}{|s|}e^{-j2\pi(f_1-\nu)d/s}\sum_{\alpha\in A} S^{\alpha}_{xx^{(*)}}\left(\frac{f_1-\nu}{s}\right)\delta\Big(f_2 -_{(-)}\Big(\alpha - \frac{f_1-\nu}{s}\Big)\Big) \tag{10}$$

which has support contained in lines with nonunit slope. Under the so-called *narrow-band condition* $BT \ll c/|v|$, with B bandwidth of $x(t)$, T data-record length, v relative radial speed, and c medium propagation speed, the time-scale factor s can be approximated with 1 and $y(t)$ and $x(t)$ can be modeled as jointly ACS. The adoption of large data-record lengths T in order to obtain satisfactory performance in the presence of low signal-to-noise ratio (SNR) or signal-to-interference ratio (SIR) requires the adoption of the SC model instead of the ACS one. This leads to significant performance improvement in parameter estimation of moving sources [3].

In the case of constant relative radial acceleration between transmitter and receiver, $D(t)$ is a quadratic function of t. Under the narrow band conditions $BT \ll c/|v|$ and $BT^2 \ll 2c/|a|$, where v is the radial speed at $t = 0$ and a is the radial acceleration, the signal $y(t)$ can be written as

$$y(t) = b\, x(t-d)\, e^{j2\pi\nu t}\, e^{j\pi\gamma t^2} \tag{11}$$

where γ is the chirp rate. In such a case, the received signal is *generalized almost cyclostationary (GACS)*. That is, its autocorrelation function is an almost-periodic function of time with both frequencies and coefficients depending on the lag parameter τ:

$$\mathrm{E}\Big\{y(t+\tau)\, y^*(t)\Big\} = \sum_{\alpha \in A} \Big[|b|^2\, R^{\alpha}_{xx^*}(\tau)\, e^{j2\pi\nu\tau}\, e^{j\pi\gamma\tau^2}\, e^{-j2\pi\alpha d}\Big]\, e^{j2\pi[\alpha+\gamma\tau]t}\,. \tag{12}$$

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On a sufficient condition for a set to be pluriregular

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Omówię pewien warunek, którego zachodzenie gwarantuje pluriregularność zbioru. Będę go nazywał warunkiem (Λ). Istnieją różne kryteria pluriregularności zbioru zwartego w $\mathbb{C}^N$. Jednym z najbardziej znanych jest kryterium osiągalności semianalitycznej Pleśniaka. Dzięki warunkowi (Λ) możliwe jest w pewnych sytuacjach wykazanie pluriregularności zbiorów, dla których kryterium osiągalności semianalitycznej nie zachodzi. Podam różne przykłady, w tym związane z szeregami uogólnionymi. Ponadto przypomnę pewien problem Sadullaeva i jego konsekwencje w kontekście warunku (Λ). W miarę możliwości czasowych podam też pewne związki z aproksymacją wielomianową funkcji.

Bornologies, lattices and locally small spaces

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We consider the category **GTS** of generalized topological spaces and their strictly continuous mappings.
Theorem 1. The construct **GTS** is topological. In particular, it is a complete and co-complete category.

A *gts* (X, Op_X, Cov_X) is called **partially topological** if Op_X is a topology, and **topological** if $Cov_X = \mathcal{P}(Op_X)$. We call a subset K of a generalized topological space X **small** if for each admissible covering $\mathcal{U}$ of any open U, the set $K \cap U$ is covered by finitely many members of $\mathcal{U}$. (We say that $\mathcal{U}$ is **essentially finite** on K in such a situation.) A *gts* is **locally small** if there is an admissible covering of the whole space by small open subsets. Locally small *gts*-es form a full subcategory **LSS** of **GTS**.

Proposition 2. The construct **LSS** has concrete finite products and concrete direct sums.

Define the category **Sublat** as follows: objects are pairs $(X, \mathcal{L})$, where X is any set and $\mathcal{L}$ is a sublattice of $\mathcal{P}(X)$ containing the empty set and covering the set X, morphisms are mappings $f : X \to Y$ such that $\mathcal{L}_X \preceq f^{-1}(\mathcal{L}_Y)$ and $f^{-1}(\mathcal{L}_Y) \cap_1 \mathcal{L}_X \subseteq \mathcal{L}_X$.

Theorem 3. The categories **LSS** and **Sublat** are isomorphic.

A **bornology** $\mathcal{B}$ on a set X is an ideal in $\mathcal{P}(X)$ containing every singleton. The pair $(X, \mathcal{B})$ is then called a **bornological set**, and each member of $\mathcal{B}$ a **bounded set**. A mapping between bornological sets is called **bounded** if it maps bounded sets onto bounded sets.

Proposition 4. The family of small sets Sm_X of a *gts* X is always a bornology. In a locally small *gts* each small set is contained in a small open set, hence the family $Smop_X$ of small open sets is an (open) basis of the bornology Sm_X.

A **bornological universe** is a triple $(X, \tau, \mathcal{B})$, where τ is a topology, and $\mathcal{B}$ is a bornology. By **Ubor** we denote the category of bornological universes with continuous bounded mappings.

Theorem 4. The category $\mathbf{LSS}_{pt}$ of partially topological locally small spaces is isomorphic to **UBorOB**, the full subcategory in **UBor** of bornological universes having open bases.

On the Richberg Theorem

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We give a new proof of Richberg Theorem about the smoothing of continuous plurisubharmonic functions. The main tool is the Cafarelli-Kohn-Nirenberg-Spruck theorem.

On the Goldie dimension

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The Goldie dimension of a module M is definied as the supremum of all cardinalities λ such that M contains the direct sum of λ non-zero submodules. It can be considered as a generalization of the linear dimension from linear spaces to modules.

This invariant was introduced by Goldie in his studies of the structure of noncommutative noetherian rings more than 50 years ago and became one of the most important tools in the theory of rings and modules. The aim o the talk is to survey some old and new results and studies related to that dimension.

Quadratic automorphisms

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Centralnym problemem dotyczącym automorfizmów wielomianowych przestrzeni wektorowej $\mathbb{k}^n$ ($\mathbb{k}$ - ciało charakterystyki zero) jest następujące pytanie:

$(TGP)_n$ Czy każdy automorfizm przestrzeni $\mathbb{k}^n$ jest *strukturalnie prosty*, tzn. czy jest złożeniem skończenie wielu automorfizmów afinicznych oraz automorfizmów postaci $(x_1, \ldots, x_n) \longmapsto, (x_1, \ldots, x_{i-1}, x_i + f, x_{i+1}, \ldots, x_n), f \in \mathbb{k}[X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n]$?

Wiadomo, że odpowiedź na nie jest pozytywna dla $n = 2$ (klasyczne tw. Junga - van der Kulka) oraz negatywna dla $n = 3$ (głębokie wyniki Szestakova i Umirbaeva). Pytanie pozostaje całkowicie otwarte dla $n \geq 4$.

Z pytaniem $(TGP)_n$ ściśle wiąże się jego zawężenie:

$(QAP)_n$ Czy każdy automorfizm wielomianowy stopnia drugiego przestrzeni $\mathbb{k}^n$ jest strukturalnie prosty?

Wiadomo, że tak jest dla $n \leq 4$ (Meisters i Olech) oraz znane są pewne szczególne klasy strukturalnie prostych automorfizmów kwadratowych działających w wyższych wymiarach. W większości tych przykładów strukturalną prostotę uzyskuje się dowodząc mocniejszej własności, mianowicie *liniowej triangularyzowalności* (jak np. dla kwadratowych quasi - translacji).

Głównym celem referatu jest zwrócenie uwagi na metody teorii macierzy, które pozwalają niezwykle prosto uzyskać pewne znane rezultaty (np. liniową triangularyzowalność kwadratowych quasi-translacji) oraz wskazać nową klasę automorfizmów kwadratowych liniowo triangularyzowalnych. Zostaną także zaprezentowane inne problemy dotyczące $(QAP)_n$.

Boundary value problem for the second order impulsive delay differential equations

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The aim of the paper is to present result on the existence and the uniqueness of the solutions of the following boundary value problem for second order delay differential systems with impulses at fixed points:

$$\begin{aligned} & x''(t) = f(t, x_t), \quad t \in J' = J \setminus \{t_1, ..., t_p\}, \\ & \triangle x(t_k) = I_{0k}\left(x(t_k), x'(t_k)\right), \quad k = 1, ..., p, \\ & \triangle x'(t_k) = I_{1k}\left(x(t_k), x'(t_k)\right), \quad k = 1, ..., p, \\ & x_0 = \phi, \quad x'(T) = \beta x'(0), \beta > 1, \end{aligned} \tag{1}$$

where $J = [0, T], T > 0, 0 = t_0 < t_1 < ... < t_p < t_{p+1} = T$, $f : J \times PC([-\tau, 0], \mathbb{R}^n) \to \mathbb{R}^n$ is given function, $\phi \in PC([-\tau, 0], \mathbb{R}^n), \tau > 0$,

$$\begin{aligned} PC([-\tau, 0], \mathbb{R}^n) = \big\{ & x : [-\tau, 0] \to \mathbb{R}^n : x(t^-) = x(t) \text{ for all } t \in (-\tau, 0], x(t^+) \\ & \text{exists for all } t \in [-\tau, 0), \text{ and } x(t^+) = x(t) \text{ for all} \\ & \text{but at most a finite number of points } t \in [-\tau, 0) \big\}. \end{aligned}$$

For any function $x : [-\tau, T] \to \mathbb{R}^n$ and any $t \in J$, we let x_t denote the function $x_t : [-\tau, 0] \to \mathbb{R}^n$ defined by

$$x_t(s) = x(t + s), \qquad s \in [-\tau, 0].$$

$\triangle x(t_k), \triangle x'(t_k)$ denote the jump of $x(t), x'(t)$ at $t = t_k$, i.e.

$$\begin{aligned} \triangle x(t_k) &= x(t_k^+) - x(t_k^-), \\ \triangle x'(t_k) &= x'(t_k^+) - x'(t_k^-), \end{aligned}$$

where $x(t_k^+), x'(t_k^+), x(t_k^-), x'(t_k^-)$ represent the right and left limits of $x(t), x'(t)$ at $t = t_k$, respectively, $I_{0k}, I_{1k} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$.

In order to define the concept of solution for (1) we introduce the following sets of functions:

$$PC[J,\mathbb{R}^n] = \{x : J \to \mathbb{R}^n : x \text{ is continuous at } t \neq t_k,$$
$$\text{left continuous at } t = t_k, \text{ and } x(t_k^+) \text{ exists }, k = 1, ..., p\Big\},$$

$$PC^1[J,\mathbb{R}^n] = \{x \in PC[J,\mathbb{R}^n] : x'(t) \text{ exists and is continuous at } t \neq t_k,$$
$$\text{and } x'(t_k^+), x'(t_k^-) \text{ exist for } k = 1, ..., p\Big\}.$$

Denote $C^\star = PC^1\left([-\tau, T], \mathbb{R}^n\right) \cap C^2\left(J', \mathbb{R}^n\right).$

A function $x \in C^\star$ is said to be a solution of (1) if x satisfies (1) for $t \in J$. Function $x \in C^\star$ is a solution of (1) if and only if x is a solution of some integral equation. First we transform the problem (1) into fixed point problem. Then using the Banach fixed point theorem we obtain the main result on the existence and uniqueness of the solutions of the problem (1).

The existence results for the boundary value problem for second order delay differential equations of the above type without impulsive conditions have been studied in [1].

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A few remarks about the linear capacity of algebraic sets

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Niech V będzie skończenie wymiarową przestrzenią wektorową nad ciałem $\mathbb{F}$, wyposażoną w topologię Zariskiego. Pojemnością liniową zbioru $E \subseteq V$ nazywa się

$$\sup \left\{ \dim L : \, L \text{ jest podprzestrzenią liniową przestrzeni } V, \, L \subseteq \overline{E}, \, L \cap E \neq \emptyset \right\}.$$

Pojemność liniową zbioru E będziemy oznaczać przez $\lambda(E)$.

O pojemności liniowej mówi wiele nietrywialnych twierdzeń. Szczególnie interesujące wydają się te z nich, które są związane z teorią macierzy. Klasyczny wynik Dieudonnégo dotyczący podprzestrzeni osobliwych w przestrzeni macierzy kwadratowych można np. sformułować w następujący sposób:

$$\lambda(\{A \in \mathcal{M}_n(\mathbb{F}) : \, \det(A) = 0\}) = n(n-1).$$

Ogólne twierdzenie Gerstenhabera o podprzestrzeniach liniowych zawartych w stożku nilpotentnym mówi z kolei, że jeśli $\mathrm{char}(\mathbb{F}) = 0$ oraz macierz $A \in \mathcal{M}_n(\mathbb{F})$ jest nilpotentna, to

$$\lambda(\{U^{-1}AU : \, U \in \mathcal{M}_n(\mathbb{F}), \, \det(U) \neq 0\}) = \frac{1}{2}\big(n^2 - \sum_{j=0}^{\infty}(r_A(j) - r_A(j+1))^2\big),$$

gdzie $r_A(j) = \mathrm{rank}(A^j)$. Uogólnienie wyniku Dieudonnégo pochodzące od Flandersa i Meshulama głosi w końcu, że jeśli $r \in \{0, \ldots, n\}$, to

$$\lambda(\{A \in \mathcal{M}_n(\mathbb{F}) : \, \mathrm{rank}(A) \leq r\}) = nr.$$

W referacie omówimy podstawowe własności pojemności liniowej oraz wybrane twierdzenia geometrii algebraicznej i teorii macierzy dotyczące tej pojemności. Zaprezentujemy ponadto kilka nowych spostrzeżeń i pytań odnoszących się do pojemności liniowej zbiorów macierzy.

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Credit risk assessment models

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Istotą wyceny ryzyka kredytowego jest wyznaczenie rozkładu potencjalnych strat z działalności kredytowej. Dysponując takim rozkładem, można obliczyć oczekiwaną wielkość straty oraz rozproszenie wielkości potencjalnych strat wokół wartości średniej. Parametry rozkładu mają istotną wartość informacyjną dla instytucji kredytowej, np. banku, ponieważ umożliwiają ustalenie odpowiedniej ceny kredytu (stopy oprocentowania), co następnie pozwala maksymalizować zyski z działalności kredytowej. W referacie omówimy wybrane modele wyceny ryzyka kredytowego i przedstawimy proste przykłady zastosowania niektórych z nich.

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Some observations concerning reducibility of quadrinomials

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In a recent paper, Jankauskas proved some interesting results concerning the reducibility of quadrinomials of the form $f(4,x)$, where $f(a,x) = x^n + x^m + x^k + a$. He also obtained some examples of reducible quadrinomials $f(a,x)$ with $a \in \mathbb{Z}$, such that all the irreducible factors of $f(a,x)$ are of degree ≥ 3.

During the talk we present results concerning a more systematic approach to the problem and ask about reducibility of $f(a,x)$ with $a \in \mathbb{Q}$. In particular by computing the set of rational points on some genus two curves we characterize in several cases all quadrinomials $f(a,x)$ with degree ≤ 6 and divisible by a quadratic polynomial. We also give further examples of reducible $f(a,x)$, $a \in \mathbb{Q}$, such that all irreducible factors are of degree ≥ 3. This talk is based on a joint work with A. Bremner (Arizona State University).

Geometry of polynomial mappings at infinity

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For the given polynomial map $F : \mathbb{C}^n \to \mathbb{C}^n$ we construct a pseudomanifold N_F. In the case $n = 2$ we will show that the map F with non-vanishing jacobian is not proper iff the homology or the intersection homology of the set N_F is nontrivial.

Wallman-Frink extensions of generalized topological spaces

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Let us work on the subtheory $\mathbf{Z} = Z +$ [Axioms of Logic] of the theory $\mathbf{ZF} = ZF +$ [Axioms of Logic] where ZF is the standard Zermelo-Fraenkel system of axioms (cf.[3]). The notion of a weakly normal gts introduced by A. Piękosz in (cf.[4]) is so strictly relevant to the concept of a Wallman base (called also a normal base)(cf.[1] and [5]) that it is reasonable to introduce generalized topological spaces (in abbr. gtses) induced by complete closed bases of usual topological spaces in order to start in $\mathbf{Z}$ a theory of Wallman-Frink extensions of gtses. Namely, if $\mathcal{C}$ is a base for the closed sets of a topological space X such that $\mathcal{C}$ is a ring of sets and $\emptyset, X \in \mathcal{C}$, let us say that $\mathcal{C}$ is a complete closed base for X, while the triple $\langle X,\ \mathrm{Op}_{\mathcal{C}}(X),\ \mathrm{Cov}_{\mathcal{C}}(X)\rangle$ is the gts induced by $\mathcal{C}$ where $\mathrm{Op}_{\mathcal{C}}(X) = \{X \setminus C : C \in \mathcal{C}\}$, whereas $\mathrm{Cov}_{\mathcal{C}}(X)$ is the collection of all essentially finite families of members of $\mathrm{Op}_{\mathcal{C}}(X)$. It occurs that the gts induced by a complete closed base $\mathcal{C}$ is weakly normal if and only if $\mathcal{C}$ is a Wallman base. The ultrafilter theorem (**UFT**) states that, on an arbitrary set, every filter is contained in an ultrafilter (cf. [2]). I prove that the following theorem holds:

Theorem It is valid in $\mathbf{Z}$ that **UFT** is equivalent with the statement: for every semi-normal in Frink's sense space X and for every Wallman base $\mathcal{C}$ of X, the Wallman-Frink extension $w_{\mathcal{C}}X$ of X generated by $\mathcal{C}$ is compact.

Therefore, although **UFT** is weaker than **AC**, the theory $\mathbf{Z} + \mathbf{UFT}$ is convenient for investigations of compactifications of Wallman type. I offer such necessary and sufficient conditions for a mapping to be continuously extendable over a compactification of Wallman type that are related to the concept of strictly continuous mappings of gtses.

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Industrial mathematics at "Poznańskie Praktyki Badawcze"

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Poznańskie Praktyki Badawcze (PPB) to czerpiące z najlepszych brytyjskich wzorców warsztaty, promujące współpracę pomiędzy środowiskiem naukowym a firmami i instytucjami. Warsztaty oparte są na założeniach Industrial Mathemathics – dziedziny badań wywodzącej się z Oxfordu. Ich celem jest usprawnianie procesów, produktów i usług w firmach i instytucjach z wykorzystaniem narzędzi matematycznych i informatycznych. Podobne wydarzenia są organizowane z powodzeniem na całym świecie od kilkudziesięciu lat i przyciągają najlepszych studentów, doktorantów i pracowników naukowych z uznanych ośrodków akademickich. Podczas tego wprowadzenia podane będą tematy realizowanych projektów oraz to, czym w praktyce jest matematyka przemysłowa.