Workshop on Modern Applied Mathematics PK 2014

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Book of Abstracts

Workshop on Modern Applied Mathematics
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prof. dr hab. Wojciech Gajda (Adam Mickiewicz University in Poznań)
prof. dr hab. Tadeusz Stanisz (Cracow University of Economics)

Contents

Introduction

Workshop on Modern Applied Mathematics PK 2014 is an annual conference organized by the Institute of Mathematics of the Faculty of Physics, Mathematics and Computer Science, Tadeusz Kościuszko Cracow University of Technology.

The Conference aims to present new results, to promote and to bring together researchers in the various research areas of mathematics and influence more cooperation among scientists working in mathematics. This conference will provide a unique forum for exchanging ideas and in-depth discussions on different aspects and different branches of mathematics.

You can find detailed information about the conference at

`www.wmam.pk.edu.pl`

Extended versions of the papers presented at the conference may be published in Technical Transactions (`www.czasopismotechniczne.pl`) (a scientific journal from the list B of the Polish Ministry of Science and Higher Education) on the usual terms. Every participant is invited to submit an article.

I would like to thank professors Sławomir Cynk, Antoni Leon Dawidowicz, Wojciech Gajda and Tadeusz Stanisz for accepting invitation to give a lecture during the conference and all participants for interest in the third edition of our conference and scientific research in the area of mathematics.

The conference is supported this year by Municipality of Krakow (Urząd Miasta Krakowa) (`www.krakow.pl/english/business/8921,glowna.html`, `www.krakow.pl/english/`) and is under the media patronage of

Welcome to Cracow & Małopolska (www.welcome.com.pl). I would like to thank them for their help in the organization of the conference.

I would like to express my thanks to Board of Directors and the Administration of the Institute of Mathematics as well as the staff of the Institute for their friendliness and support for the conference organization.

My special thanks go to Mr Konrad Koterla for the website maintenance and Mr Jakub Szotek for help in text translation.

On the behalf of the organizing committee
Grzegorz Gancarzewicz

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Abstracts

The existence of solution for differential inclusions with the operator of $\mathbf{p(x)}$-Laplacian

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We consider a nonlinear elliptic differential inclusions with $p(x)$-Laplacian and with Dirichlet boundary condition. These are the so-called hemivariational inequalities and they are derived with the help of subdifferential in the sense of Clarke.

In particular, we will consider the following problem

$$\begin{cases} -\Delta_{p(x)}u(x) - \lambda|u(x)|^{p(x)-2}u(x) \in \partial j(x, u(x)) & \text{a.e. in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with the smooth boundary $\partial\Omega$, $p : \overline{\Omega} \to \mathbb{R}$ is a continuous function satisfying $p(x) > 1$ for all $x \in \Omega$. The function $j(x, t)$ is locally Lipschitz in t-variable and measurable in x-variable. By $\partial j(x, t)$ we denote the subdifferential with respect to the t-variable in the sense of Clarke. The operator

$$\Delta_{p(x)}u := \operatorname{div}\big(|\nabla u|^{p(x)-2}\nabla u\big)$$

is the so-called $p(x)$-Laplacian.

We provide the necessary conditions for the existence of a solution to problem (1) in the situation when λ change the sign. By using the Ekeland variational principle and the properties of variational Sobolev spaces, we establish conditions which ensure the existence of a solution for our problem.

References

[1] S.Barnaś, Existence result for hemivariational inequality involving $p(x)$–Laplacian, *Opuscula Mathematica* **32**: 439–454 (2012).

[2] S.Barnaś, Existence result for differential inclusion with $p(x)$–Laplacian, *Schedae Informaticae* **21**: 41-55 (2012).

[3] S. Barnaś, Existence of a nontrival solution for Dirichlet problem involving $p(x)$– Laplacian, *Discussiones Mathematicae Differential Inclusions, Control and Optimization* **34**: 15–39 (2014).

A TRANSITIVE DENDRITE MAP WITH ZERO ENTROPY

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It is a result of Blokh that every transitive map of a topological tree has positive entropy and a dense set of periodic points. It was expected by some that these results generalize to maps of dendrites.

Recently, Hoehn and Mouron introduced a family of maps on the Ważewski universal dendrite which are topologically weak mixing but not mixing. We modify their construction to show that there also exists such a map with zero topological entropy. This answers the question of Baldwin and provides a new example of a transitive dendrite map whose periodic points are not dense.

This is joint work with Fryderyk Falniowski and Dominik Kwietniak.

The Perron–Frobenius Theorem and its Applications in PageRank

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The Perron–Frobenius theorem, which was firstly proven by Oskar Perron in 1907 and later extended by Georg Frobenius in 1912 asserts that a real nonnegative square matrix has a unique largest real eigenvalue and that the eigenvector corresponding to it has strictly positive components and that this eigenvector is stochastic. This theorem has a wide variety of applications: from probability theory, through economics, demography and rankings to (according to Larry Page and Sergey Brin's idea in 1996) Internet search engines. The main aim is to present this theorem with one of its most popular proofs (using the Brouwer fixed point theorem) and to explain the idea of one of its applications – PageRank, an algorithm used nowadays by Google Search.

References

[1] M. Pollicott and M. Yuri, Dynamical Systems and Ergodic Theory, *London Mathematical Society*, Student Texts **40**: 23–25 (1998).

[2] S. Sternberg, *Lecture 12: The Perron–Freobnius theorem*, available here: http://www.math.harvard.edu/library/sternberg/slides/1180912pf.pdf.

[3] C.R. MacCluer, The Many Proofs and Appications of Perron's Theorem, *SIAM Review* Vol 42, No.3: 487–498 (2000), available here: http://epubs.siam.org/doi/pdf/10.1137/S0036144599359449.

[4] S. Brin, L. Page, *The Anatomy of a Large–Scale Hypertextual Web Search Engine*, available here: http://infolab.stanford.edu/ backrub/google.html.

[5] M. Agrawal, *The Perron–Frobenius Theorem and Google's PageRank Algorithm* (2010), available here: https://mohitagrawal.files.wordpress.com/2010/02/presentation.pdf.

Differential operators of Calabi–Yau type

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I will report on a long lasting search for linear differential operators of order four with very special properties called Calabi–Yau type. Motivated by Mirror Symmetry Conjecture these operators are expected to be related to one parameter families of Calabi–Yau threefolds and can be considered as "higer–dimensional" counterpart of the Picard–Fuchs operator discovered by Euler.

It is a joint project with Duco van Straten (Mainz).

References

[1] S. Cynk, D. van Straten, Calabi-Yau conifold expansions, *Arithmetic and geometry of K3 surfaces and Calabi-Yau threefolds*, 499–515, Fields Inst. Commun., 67, Springer, New York, 2013.

What is the biomathematics or why the delayed differential equations are applied in biology and medicine?

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The classical theory of differential equations is the results of requirement of physics. So, physical system reacts to stimulus at once. The biological system reacts to stimulus after certain time. Hence, for description of biological systems it is better to use the equation with delayed argument.

Measures on spectra of commutative rings

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Let R be a commutative ring with identity. In the talk, we will discuss relationships between measures on R and measures on $\mathrm{Spec}(R)$ (by measures we mean σ-additive positive measures). We will also provide some examples related to polynomial rings and rings of continuous functions.

References

[1] M. F. Atiyah, I. G. MacDonald, *Introduction to Commutative Algebra*, Westview Press, 1994.

[2] H. Federer, *Geometric Measure Theory*, Springer-Verlag, 1996.

Geometric representations of Galois group

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In the talk, I will discuss some problems of contemporary arithmetic geometry. The talk will be about algebraic varieties defined over an arbitrary field K, their etale cohomology groups and actions of the absolute Galois group. The most interesting problems concern the case where K is the field of rational numbers.

The central limit theorems and their applications for the mean in periodic non-stationary, heavy tailed and long memory time series

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New and a more general concept of dependence in time series - the weak dependence was introduced by Doukhan in 1999 [1]. This kind of dependence gives tools for the analysis of statistical procedures with very general data generating processes. One of such statistical procedures is subsampling.
Unfortunately without knowledge about existence of non-degenerated asymptotic distribution for estimated parameters the subsampling procedure can't be used.
In the presentation the model which simultaneously deals with three features of time series: periodic non-stationarity, heavy tails and long memory will be introduced. The model investigated in the presentation can be considered as an extension of the results by [4].
The main goal of the talk is to investigate the asymptotic behavior of the vectors of sample means for the periodic non-stationary, heavy tailed and long memory process in both cases of heavy tails: stable and GED. Moreover, application of the subsampling method to estimate the vector of the means will be shown. Weak dependence conditions allow to achieve positive results.

References

[1] P. Doukhan, J. Dedecker, G. Lang, J. Leon, S. Louhichi, C. Prieur, *Weak Dependence: With Examples and Applications*, Springer-Verlag, 2008.

[2] J. Leśkow, L. Lenart, R. Synowiecki, Subsampling in estimation of autocovariance for PC time series, *J. Time Ser. Anal.* **29**: 9995–1018 (2008).

[3] A. Jach, T. McElroy, D. Politis, Subsampling inference for the mean of heavy-tailed long memory time series, *J. Time Ser. Anal.* **33**: 96–111 (2012).

[4] D. Politis, J. Romano, M. Wolf, *Subsampling*, Springer-Verlag, New York, 1999.

A degree sum condition on hamiltonian cycle or on hamiltonian cycle through specified edges

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We consider only simple graphs i.e. finite graphs without loops and multiple edges. For a simple graph G we present degree sum conditions for couples or triplets of independent vertices under which the graph G is hamiltonian and we give their analogs for hamiltonian cycles through specified sets of independent edges.

Censored Linear Regression Models for Irregularly Observed Longitudinal Data using the Multivariate-t Distribution

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In acquired immunodeficiency syndrome (AIDS) studies it is quite common to observe viral load measurements collected irregularly over time. Moreover, these measurements can be subjected to some upper and/or lower detection limits depending on the quantification assays. A complication arises when these continuous repeated measures have a heavy-tailed behavior. For such data structures, we propose a robust structure for a censored linear model based on the multivariate Students t-distribution. To compensate for the autocorrelation existing among irregularly observed measures, a damped exponential correlation structure is employed. An efficient expectation maximization type algorithm is developed for computing the maximum likelihood estimates, obtaining as a by-product the standard errors of the fixed effects and the log-likelihood function. The proposed algorithm uses closed-form expressions at the E-step that rely on formulas for the mean and variance of a truncated multivariate Students t-distribution. The methodology is illustrated through an application to a Human Immunodeficiency Virus-AIDS (HIV-AIDS) study and several simulation studies.

Balanced diet algorithm

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During the lecture we will present the project from Poznan Research Workshop - the algorithm for generating balanced diet. The study aimed to obtain a mathematical model for the problem of balanced diets and to create menu from the meal and products base. The outcome was an application that after reading the caloric intake, it returns a balanced menu for the entire month in which they ensure the necessary number of calories, proteins, fats and carbohydrates. Additional terms and conditions that meet the model are: the exclusion of unwanted products, dietary diversity and reduced number of necessary products to n per week. The presented algorithm is based on the use of kD-tree (k-dimensional tree) and greedy methods.

The degree of approximation of functions by Szász-Mirakjan type operators

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The aim of the talk is to present theorems giving the degree of approximation of functions by modified Szász-Mirakjan operators defined as follows

$$A_n^{\nu}(f;x) = \begin{cases} \sum_{k=0}^{\infty} p_{n,k}^{\nu}(x) f(\frac{2k}{n}), & x > 0; \\ f(0), & x = 0 \end{cases}$$

and

$$p_{n,k}^{\nu}(x) = \frac{1}{I_{\nu}(nx)} \frac{(nx)^{2k+\nu}}{2^{2k+\nu} k! \Gamma(k+\nu+1)},$$

where Γ is the gamma function and I_{ν} stands for the modified Bessel function, i.e.

$$I_{\nu}(z) = \sum_{k=0}^{\infty} \frac{z^{2k+\nu}}{2^{2k+\nu} k! \Gamma(k+\nu+1)}.$$

We study the estimation error of our operators in weighted spaces.

References

[1] M. Heilmann, Direct and converse results for operators of Baskakov-Durrmeyer type, *Approx. Theory Appl.* **5**(1): 105–127 (1989).

[2] M. Herzog, Approximation of functions from exponential weight spaces by operators of Szász-Mirakyan type, *Comment. Math.* **43**(1): 77–94 (2003).

[3] L. Rempulska, A. Thiel, Approximation of functions by certain nonlinear integral operators, *Lith. Math. J.* **48**(4): 451–462 (2008).

A survey on known values and bounds on the Shannon capacity

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We present exact values and bounds on the Shannon capacity for different classes of graphs, for example for regular graphs and Kneser graphs. Additionally, we show a relation between Ramsey numbers and Shannon capacity.

References

[1] C. Shannon, The zero error capacity of a noisy channel, *IEEE Trans. Inform. Theory* **IT–2**: 8–19 (1956).

[2] J. Körner and A. Orlitsky, Zero-error information theory, *IEEE Trans. Inform. Theory* **44.6**: 2207–2229 (1998).

[3] J. Körner and I. Csiszár, *Information Theory*, Cambridge University Press, New York, 2011.

Banach-Stone Theorem

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Let X be a compact topological space. We will denote by $\mathcal{C}(X)$ the Banach algebra of all continuous functions $f : X \longrightarrow \mathbb{R}$ (equipped with the supremum norm $\|\cdot\|_\infty$). This algebra is partially ordered by "pointwise inequality" $\leqslant$. The classical Banach-Stone theorem says that if X and Y are compact topological spaces and $\Phi : \mathcal{C}(X) \longrightarrow \mathcal{C}(Y)$ is an isometric linear isomorphism, then there exist a homeomorphism $\chi : Y \longrightarrow X$ and a function $g \in \mathcal{C}(Y)$ such that

$$\begin{cases} |g(y)| = 1 \text{ for all } y \in Y, \\ \Phi(f) = (f \circ \chi)g \text{ for all } f \in \mathcal{C}(X). \end{cases}$$

Let $\boldsymbol{\lambda}$ denote a constant function equal to a scalar $\lambda \in \mathbb{R}$. In the talk, we will present an elementary proof (based on [2]) of the theorem stated below and derive the Banach-Stone theorem as an easy consequence.

Theorem. *If X and Y are compact topological spaces, then for a linear isomorphism $\Psi : \mathcal{C}(X) \longrightarrow \mathcal{C}(Y)$ with $\Psi(\mathbf{1}) = \mathbf{1}$ the following conditions are equivalent:*

(1) $\forall\, f \in \mathcal{C}(X) :\ f \geqslant \mathbf{0} \Longleftrightarrow \Psi(f) \geqslant \mathbf{0}$,

(2) $\forall\, f \in \mathcal{C}(X) :\ \|\Psi(f)\|_\infty = \|f\|_\infty$,

(3) $\forall\, f_1, f_2 \in \mathcal{C}(X) :\ \Psi(f_1 f_2) = \Psi(f_1)\Psi(f_2)$,

(4) there exists a homeomorphism $\chi : Y \longrightarrow X$ such that $\Psi(f) = f \circ \chi$ for all $f \in \mathcal{C}(X)$.

References

[1] M. I. Garrido and J. A. Jaramillo, Variations on the Banach-Stone theorem, *Extr. Math.* **17**, No. 3: 351–383 (2002).

[2] E. de Jonge and A. C. M. van Rooij, *Introduction to Riesz spaces*, Mathematical Centre Tracts 78, Amsterdam: Mathematisch Centrum, 1977.

Methods and problems in analysis of large data sets

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In the talk, we will use the example of cell movements to discuss analysis of large data sets. The talk will be divided into three parts. In the first part a general description of the data set which corresponds to the cell movements will be provided. In the second part, we will present the image analysis methods applied to this set. The last part will deal with some issues related to signal analysis techniques and problems that can be encountered when working on such data sets.

Approximation by some positive linear operators associated with the Hermite polynomials

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We introduce the class of operators G_n^α, $n \in \mathbb{N} := \{1, 2, 3, \ldots\}$, $\alpha \geq 0$, given by the formula

$$G_n^\alpha(f; x) = e^{-(nx+\alpha x^2)} \sum_{k=0}^{\infty} \frac{x^k}{k!} H_k(n, \alpha) f\left(\frac{k}{n}\right), \quad x \in \mathbb{R}_0 := [0, \infty),$$

where H_k is the two variable Hermite polynomial defined by

$$H_k(n, \alpha) = k! \sum_{s=0}^{\left[\frac{k}{2}\right]} \frac{n^{k-2s}\alpha^s}{(k-2s)!s!}.$$

The operators G_n^α are linear and positive, and extend the well-known Szász-Mirakjan operators.

We study approximation properties of G_n^α for real-valued functions f continuous and bounded on $\mathbb{R}_0$. We present direct approximation theorems and the Voronovskaya type theorem for these operators. The results are given in one and two dimensions.

References

[1] R. A. DeVore, G. G. Lorentz, *Constructive Approximation,* Springer-Verlag, Berlin, 1993.

On sets of generators for finite groups

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The talk will be about numbers of generators for finite groups. Special attention will be paid to groups with sets of generators of fixed cardinalities (groups with the basis property), and their generalizations. Some examples will also be presented.

References

[1]P. Apisa, B. Klopsch, A generalization of the Burnside Basis Theorem, *J. Algebra* **400**: 8–16 (2014).

[2] O. Artemowicz, A. Piękosz, *Algebra*, PK, Kraków 2010.

[3] C. Bagiński, *Wstęp do Teorii Grup,* Script, Warszawa 2002.

[4] J. Krempa, A. Stocka, On some sets of generators of finite groups, *J. Algebra* **405**: 122–134 (2014).

[5] J. Krempa, A. Stocka, Corrigendum to 'On some sets of generators of finite groups', *J. Algebra* **408**: 61–62 (2014).

[6] J. Krempa, A. Stocka, On sets of pp-generators of finite groups, *Bull. Austral. Math. Soc.* (in print).

[7] J. McDougall-Bagnall, M. Quick, Groups with the basis property, *J. Algebra* **346** 332–339 (2011).

COUNTEREXAMPLES TO THE $I^{(3)} \subseteq I^2$ CONTAINMENT

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Symbolic powers of ideals and containment relations between them and ordinary powers is a subject of current interest in algebraic geometry and commutative algebra. Our research is motivated by the following conjecture due to Harbourne and Huneke:

$$I^{(m)} \subseteq I^r \quad \text{holds for} \quad m \geq nr - (n-1),$$

where I is a homogeneous ideal in $(n+1)$ variables ([3], Conjecture 4.1.1.).

The simplest containment of that kind is

$$I^{(3)} \subseteq I^2 \tag{2}$$

for a homogeneous radical ideal of points I in the projective plane $\mathbb{P}^2$. This holds true in many special cases. However, in 2013 Dumnicki, Szemberg and Tutaj-Gasińska in [2] gave a counterexample over complex numbers. Their construction does not work over the reals.

The purpose of this talk is to present constructions of real counterexamples to containment 2.

References

[1] A. Czapliński, A. Główka, M. Lampa-Baczyńska, P. Łuszcz-Świdecka, G. Malara, P. Pokora, J. Szpond: A counterexample to the containment $I^{(3)} \subseteq I^2$ over the reals, *arXiv:1310.0904v1*.

[2] M. Dumnicki, T. Szemberg, H. Tutaj-Gasinska, A counter-example to a question by Huneke and Harbourne, *J. Algebra* **393**: 24–29 (2013).

[3] B. Harbourne, C. Huneke, Are symbolic powers highly evolved?, *J. Ramanujan Math. Soc.* **28**: 311–330 (2013).

Linear Algebra as a language of the Quantum Information Theory

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Quantum Information Theory (QIT) is a young discipline which takes an effort to investigate how information stored in a quantum system can be processed. QIT presents a notion of a qubit, which is a quantum counterpart of a classical bit, that exists in a certain superposition of states "0" and "1", which is: $|\psi\rangle = a\,|0\rangle + b\,|1\rangle$, where $|a|^2 + |b|^2 = 1$. Thus, a classical bit can be considered as a special case of a qubit when $(a = 0) \oplus (b = 0)$ and the QIT can be seen as a generalization of the Classical Information Theory.

The aim of this talk is to present how Linear Algebra is used to describe quantum systems, process information with quantum algorithms and discover unexpected nature of QIT. The main part will be followed by a short introduction to core concepts in QIT and their motivation from Quantum Physics. Since quantum systems can be represented as a linear superposition of basis vectors in a complex Hilbert space, elegant Dirac notation will be introduced.

References

[1] Michael A. Nielsen, Isaac L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, 2000.

[2] Leonard Susskind, Art Friedman *Quantum Mechanics: The Theoretical Minimum*, Basic Books, 2014.

On subsampling techniques available for nonstationary, continuous time stochastic processes

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In the talk, the recent advances on subsampling techniques for nonstationary, continuous time stochastic processes will be discussed. Applications to signal processing, finance and climate data will be addressed.

On the containment hierarchy for simplicial ideals

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The containment problem for symbolic and usual powers of the ideals has been studied by many authors in recent years (see e.g.[1]).

In this talk we presents containment relations between symbolic and ordinary powers for ideals of fixed codimension skeletons (simplicial ideals) determined by arrangements of $n+1$ general hyperplanes in the n–dimensional projective space over an arbitrary field [2].

References

[1] C. Bocci, B. Harbourne, Comparing Powers and Symbolic Powers of Ideals, *J. Algebraic Geometry* **19**: 399–417 (2010).

[2] M. Lampa-Baczyńska, G. Malara, On the containment hierarchy for simplicial ideals *arXiv:1408.2472*.

Properties of the Baskakov-Durrmeyer type operators

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We consider some approximation properties of Baskakov-Durrmeyer type operators $M_n^{\alpha,a}$ defined by

$$M_n^{\alpha,a}(f;x) = n\sum_{k=0}^{\infty} W_{n,k}^a(x)\frac{1}{\Gamma(\alpha+k+1)}\int_0^{\infty} e^{-ns}(ns)^{\alpha+k}f(s)ds,\ x \geq 0,$$

where $n \in \mathbb{N},\ \alpha > -1,\ \Gamma$ is the gamma function,
and

$$W_{n,k}^a(x) = e^{-\frac{ax}{1+x}}\frac{P_k(n,a)}{k!}\frac{x^k}{(1+x)^{n+k}},\ P_k(n,a) = \sum_{i=0}^{k}\binom{k}{i}(n)_i a^{k-i},\ a \geq 0,$$

$$(n)_0 = 1,\ (n)_i = n(n+1)...(n+i-1),\ i \geq 1.$$

We establish certain direct theorems via first and second order of the modulus of continuity. Moreover we prove the Voronovskaya type theorem.
The talk is based on joint work with Eugeniusz Wachnicki.

References

[1] R. Malejki, E. Wachnicki, On the Baskakov-Durrmeyer type operators, *Comm. Math.* **54**(1): 39–49 (2014).

[2] A. F. Timan, *Theory of Approximation of Function of a Real Variable*, Moscow 1960.

Arithmetic Properties of the Sequence of Derangements

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The sequence of derangements is given by a formula $D_0 = 1, D_n = nD_{n-1} + (-1)^n, n > 1$. It is a classical object appearing in combinatorics and number theory.
For each positive integer n we have $n-1 \mid D_n$. In particular, p-adic valuation of D_n is estimated from below by p-adic valuation of $n-1$ for each prime number p. We prove that there are infinitely many prime numbers p such that $v_p(D_n) > v_p(n-1)$ for some positive integer n. Moreover, we give the description of p-adic valuation of $\frac{D_n}{n-1}$ for a given prime number p.
We show that for each positive integer k there exists polynomial $f_k \in \mathbb{Z}[X]$ such that $D_{n+k} = (n+1)...(n+k)D_n + (-1)^n f_k(n)$ for all positive integers n. Furthermore we prove that f_k has exactly $k-1$ distinct real roots, among which there is exactly one rational root $1-k$.

Integrability of geodesic flows for metrics on homogeneuos spaces of compact groups

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We find a sufficient condition for the complete integrability of the geodesic flow of the Riemannian metric on homogeneous space G/K which is induced by the invariant Riemannian metric on G. The constructed integrals are real analytic functions, polynomial in momenta. It is checked that this sufficient condition holds when G is the orthogonal group $SO(n)$ and $K = SO(n_1) \times SO(n_2) \times \cdots \times SO(n_k)$, $n_1, n_2, .., n_k \geqslant 1$, $n_1 + n_2 + \cdots + n_k = n$. For the proof we use the moment-map method developed in our paper [1] and the paper [2].

References

[1] I. V. Mykytyuk, A. Panasyuk, Bi-Poisson structures and integrability of geodesic flows on homogeneous spaces, *Transformation Groups* **9**: 289–308 (2004).

[2] V. Dragović, B. Gajić, B. Jovanović, Singular Manakov flows and geodesic flows on homogeneous spaces of $SO(N)$, *Transformation Groups* **14**: 513–530 (2009).

The mathematical model of determining the crystal stress tensor

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Each crystal can be represented using the lattice and the unit cell, since the structure is periodic. Our task was to create the algorithm, which would describe how crystal is formatting. The input were data about a unit-cell saved in Crystallography Information File. Every unit-cell consists of irreducible Brillouin zones. We can reconstruct a unit-cell by using symmetry group of crystal and the irreducible Brillouin zone. Moreover, every atom and particle can be described by using Miller index, and such description is unique.

We managed to create mathematical model in which we restricted to analysis of symmetry group of crystal, unit cell dimensions and distribution of atoms in irreducible Brillouin zone.

In the mathematical model we assumed, that crystal deformation depends only from forces between particles which is proportional to value of overlap integral. For fixed change of crystal volume, the optimal shape of unit-cell is obtained when the overlap integral value is minimal. To calculate the overlap integral value we decided to use van der Waals model. In such model, atoms are treated like a balls, where radius depends on atom type, and the particle is a union of atoms. Radius calculated in this way is called van der Waals radius. The overlap integral value is proportional to a volume of union of intersections over all distinct particles in unit-cell. Moreover, the particles are rigid.

When an ideal of an ideal is an ideal?

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In the class of associative rings the relation of being an ideal is not transitive. Rings in which this property does hold are called filial. Such rings were studied by many authors in various contexts and there are many studies concerning a description of such rings. They can be considered as an analog of extensively studied t-groups, i.e., groups in which every subnormal subgroup is a normal subgroup or as a generalization of Hamilton rings, i.e., rings in which all subrings are ideals.

The aim of the talk is to survey some old and new results related to this topic and ask some questions.

References

[1] R. R. Andruszkiewicz, E. R. Puczyłowski, On filial rings, *Portugaliea Math.* **45**: 139–149 (1988).

[2] A. D. Sands, On ideals in over-rings, *Publ. Math. Debrecen* **35**: 273–279 (1988).

[3] G. Tzintzis, A one-sided admissible ideal radical which is almost subidempotent, *Acta Math. Hung.* **49**: 307-314 (1987).

[4] S. Veldsman, Extensions and ideals of rings, *Publ. Math. Debrecen* **38**: 297–309 (1991).

Ordered sets in the Category Theory

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There are two ways in which partially ordered sets are looked at by the Category Theory. Firstly, all such sets and monotonic functions between them form a concrete category. Secondly, each ordered set is a category itself.

When subobjects are defined in the category of ordered sets, they turn out not to coincide with substructures in the sense of model theory. This fact seems to be unavoidable since the category theory notions are defined using only equality and composition. We propose a notion of an order-monic that uses also the inequality relation. Whith this new notion, the category-theoretical subobjects become exactly the same as the the substructures in the sense of model theory. A similar observation can be made for other categories of relational structures.

References

[1] M. Barr, Ch. Wells *Category Theory for Computing Science*, Prentice Hall , 1990.

A generalization of the Remez inequality

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The classical Remez inequality is the following: Suppose that $A \subset [0,\,1]$ is measurable and $|A| > 0$. Then, for each polynomial Q with $\deg Q \leq n$,

$$\|Q\|_{[0,\,1]} \leq T_n\left(\frac{2-|A|}{|A|}\right)\|Q\|_A\,,$$

where $|A|$ denotes the Lebesgue measure of A and T_n denotes the Chebyshev polynomial of degree n. Clearly, one can consider similar inequalities if the interval $[0,\,1]$ is replaced by another compact set in $\mathbb{R}^N$ and the talk will address this issue.

Generalized topology and quasi-pseudometrics

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(Joint work with Eliza Wajch.) For a gts X, the two basic bornologies on it are: the bornology $\mathbf{Sm}(X)$ of small subsets, and the bornology $\mathbf{ACB}(X)$ of relatively admissibly compact subsets. We say (for $\mathcal{S} = \mathbf{Sm}$ or $\mathbf{ACB}$) that X is $\mathcal{S}$-(quasi)-(pseudo)metrizable if the bornological universe $(X, \tau(Op_X), \mathcal{S}(X))$ is (quasi)-(pseudo)metrizable. We consider the following examples:

A) For real lines with the underlying natural topology, the following are $\mathbf{Sm}$-metrizable: $\mathbb{R}_{om}, \mathbb{R}_{slom}, \mathbb{R}_{rom}, \mathbb{R}_{st}, \mathbb{R}_{lst}, \mathbb{R}_{lom}, \mathbb{R}_{l+st}, \mathbb{R}_{l+om}$;
but not the usual topological real line $\mathbb{R}_{ut}$. Nevertheless, all of the mentioned real lines are $\mathbf{ACB}$-metrizable.

B) Consider the Sorgenfrey real lines (with the right half-open interval topology τ_S). The following gtses are both $\mathbf{ACB}$- and $\mathbf{Sm}$-quasi-metrizable: $(\mathbb{R}, EF(\tau_S, \mathbf{CB}_{\tau_{nat}}(\mathbb{R})))$, $(\mathbb{R}, EF(\tau_S, \mathbf{UB}(\mathbb{R})))$, $(\mathbb{R}, EF(\tau_S, \mathbf{LB}(\mathbb{R})))$, $(\mathbb{R}, EF(\tau_S, \mathcal{P}(\mathbb{R})))$. On the other hand, the topological gts $(\mathbb{R}, EF(\tau_S, \mathbf{FB}(\mathbb{R})))$ is neither $\mathbf{ACB}$- nor $\mathbf{Sm}$-quasi-metrizable.

C) For real lines with the underlying upper topology u, the following gtses are both $\mathbf{ACB}$- and $\mathbf{Sm}$-quasi-pseudometrizable:
$(\mathbb{R}, EF(u, \mathbf{UB}(\mathbb{R})))$, $(\mathbb{R}, EF(u, \mathbf{LB}(\mathbb{R})))$, $(\mathbb{R}, EF(u, \mathbf{CB}_{\tau_{nat}}(\mathbb{R})))$.
On the other hand, the topological gts $(\mathbb{R}, EF(u, \mathbf{FB}(\mathbb{R})))$ is not $\mathbf{Sm}$-quasi-pseudometrizable but is $\mathbf{ACB}$-quasi-pseudometrizable.

In cases of (quasi)-(pseudo)metrizability, explicit formulas for (quasi)-(pseudo)metrics are found.

Preserving some functions on matrix spaces

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Let $\mathcal{M}_{m\times n}(\mathbb{F})$ be the vector space of all $m \times n$ matrices over a field $\mathbb{F}$. A linear endomorphism $\Phi : \mathcal{M}_{m\times n}(\mathbb{F}) \longrightarrow \mathcal{M}_{m\times n}(\mathbb{F})$ is said to be a (linear) preserver of a map f defined on $\mathcal{M}_{m\times n}(\mathbb{F})$, if $f \circ \Phi = f$. Characterizing all the linear preservers of a given map can be a meaningful and nontrivial task. The earliest result concerning the linear preserver problems is due to Frobenius. He proved that a linear automorphism $\Psi : \mathcal{M}_{n\times n}(\mathbb{C}) \longrightarrow \mathcal{M}_{n\times n}(\mathbb{C})$ is a preserver of the determinant if and only if there exist matrices $U, V \in \mathcal{M}_{n\times n}(\mathbb{C})$ such that $\det(UV) = 1$ and either $\Psi(A) = UAV$ for any $A \in \mathcal{M}_{n\times n}(\mathbb{C})$, or $\Psi(A) = UA^{\mathrm{T}}V$ for any $A \in \mathcal{M}_{n\times n}(\mathbb{C})$.

In the talk, we will provide an overview of known theorems on the linear preservers and give some new remarks.

References

[1] Chi-Kwong Li and Nam-Kiu Tsing, Linear preserver problems: A brief introduction and some special techniques, *Linear Algebra Appl.* **162-164**: 217–235 (1992).

[2] S. Pierce et al. (eds.), A survey of linear preserver problems, *Linear Multilinear Algebra* **33**, No. 1-2 (1992).

Functions with separated variables

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The aim of this talk is to present the notion of function separable with respect to two variables and function separable with respect to a group of variables. We give difference criteria and differential criteria of separability.

Points at rational distances from the vertices of certain geometric objects

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We consider various problems related to finding points in $\mathbb{Q}^2$ and in $\mathbb{Q}^3$ which lie at rational distance from the vertices of some specified geometric object, for example, a square or rectangle in $\mathbb{Q}^2$, and a cube or tetrahedron in $\mathbb{Q}^3$. In particular, we ask a question whether there exist rational points in the plane which lie at rational distance from the four vertices of the *rectangle* with vertices $(0,0)$, $(0,1)$, $(a,0)$, and $(a,1)$, for $a \in \mathbb{Q}$. This problem is briefly alluded to in section D19 on p. 284 of Guy's book. We reduce this problem to the investigation of the existence of rational points on members of a certain family of algebraic curves $\mathcal{C}_{a,t}$ (depending on rational parameters a, t) and show that the set of $a \in \mathbb{Q}$ for which the set of rational points on $\mathcal{C}_{a,t}$ is infinite is dense in $\mathbb{R}$ (in the Euclidean topology).

This is joint work with Andrew Bremner (Arizona State University). (The research of the author is supported by the grant of the Polish National Science Centre no. UMO-2012/07/E/ST1/00185.)

References

[1] R. Guy, *Unsolved problems in number theory*, 3 rd. edition, Springer-Verlag, 2003.

The influence of penny shares on prices of stocks traded on Warsaw Stock Exchange

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An assumption has been made that penny shares are the reason of inconsistent price valuation in the light of commonly recognized CAPM theory. Test research is conducted in a number of variants. In variant 1 all the stocks traded on Warsaw Stock Exchange are analyzed. In the consecutive variants penny shares with denomination 0,5; 1,5; 5,0; 10,0; 12,0 and 15,0 zł are eliminated.

The results of the analysis indicate that penny shares are most of all speculative, they often generate high rate of return but are characterized by low values of financial indicators.

The research has shown that the received results are consistent with the initial assumptions.

Bornologies in bitopological spaces under ZF

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Let us suppose that $\mathcal{B}$ is a bornology in a set X and that τ_1, τ_2 are topologies in X. We say that $\mathcal{B}$ is (τ_1, τ_2)-proper if, for each $A \in \mathcal{B}$, there exists $B \in \mathcal{B}$ such that $\mathrm{cl}_{\tau_2} A \subseteq \mathrm{int}_{\tau_1} B$. The bornological biuniverse $((X, \tau_1, \tau_2), \mathcal{B})$ is called quasi-metrizable if there exists a quasi-metric d in X such that the topology τ_1 is induced by d, the topology τ_2 is induced by the conjugate d^{-1} of d, while $\mathcal{B}$ is the collection of all d-bounded sets. A subset of X can be simultaneously d-bounded and d^{-1}-unbounded. A sketch of my proof that it holds true in ZF that the bornological biuniverse $((X, \tau_1, \tau_2), \mathcal{B})$ is quasi-metrizable if and only if the bitopological space (X, τ_1, τ_2) is quasi-metrizable, while $\mathcal{B}$ is second-countable and (τ_1, τ_2)-proper will be shown. The results of mine are included in my new co-authored research article with A. Piękosz.

References

[1] S. T. Hu, Boundedness in a topological space, *J. Math. Pures. Appl.* **28**: 287–320 (1949).

[2] J. C. Kelly, Bitopological spaces, *Proc. London Math. Soc.* **13**: 71–89 (1963).

[3] A. Piękosz and E. Wajch, Quasi-metrizability of bornological biuniverses in ZF, preprint, http://arxiv.org/abs/1408.4823.

Industrial Mathematics - what is it and how to do it?

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Industrial Mathematics is a branch of mathematics in which problems are solved by mathematicians and reported by widely understood the industry or business. During the lecture we will talk about the activities of the industrial mathematics in the context of Poznan research centers. The most important event is the Poznań Research Workshop (PPB) - based on the best of British experience of workshops to promote cooperation between the scientific community and the businesses and institutions. The goal of the workshop is to solve problems in the field of industrial mathematics, and creating new or improving: processes, products and services in companies and institutions with the use of mathematical tools and computer science. Similar events are organized successfully all around the world for decades and attract the best students, phDs and professors from the best universities. During the presentation it will be given an examples of projects and what it is in practice industrial math.