# Workshop on Modern Applied Mathematics PK 2015 

## Kraków 20 - 22 November 2015



Book of Abstracts


# Workshop on Modern Applied Mathematics PK 2015 

November 20-22, 2015
Kraków, Poland

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## Introduction

Workshop on Modern Applied Mathematics PK 2015 is an annual conference on modern mathematics organized by the Institute of Mathematics of the Faculty of Physics, Mathematics and Computer Science, Tadeusz Kościuszko Cracow University of Technology.

The Conference aims to present new results, to promote and to bring together researchers in the different research areas of mathematics and influence more cooperation among scientists working in mathematics. This conference will provide a unique forum for exchanging ideas and in-depth discussions on different aspects and different branches of mathematics.

You can find detailed information about conference on the site:
www.wmam.pk.edu.pl
After the conference a special issue of Technical Transactions will be published and every participant is invited to submit an article.

I would like to thank professors Taras Banakh, Grzegorz Banaszak, Henryk Gurgul, Dorota Mozyrska, Bartosz Naskręcki and Jan Stochel for accepting invitation to give a lecture during the conference and all participants for interest in the fourth edition of our conference and scientific research in the field of mathematics.

The conference is under the media patronage of Welcome Cracow.
I would like to express my thanks to Board of Directors and the Administration of the Institute of Mathematics as well as the staff of the Institute for their friendliness and support for conference organization.

My special thanks go to Mr Konrad Koterla for the website maintenance and Mr Jakub Szotek for help in text translation.

On the behalf of the organizing committee
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## Abstracts

# Moving Block Quantile Residual Bootstrap in GARMA models 

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Generalized autoregressive moving average (GARMA) models were developed to extend the univariate Gaussian ARMA time series. In particular, such models can be applied to describe discrete time series. Our paper introduces a resampling method called Moving Block Quantile Residual Bootstrap(MBQRB) to improve inference for the parameters of the GARMA model. We provide consistency theorem for MBQRB as well as simulated and real data example related to hospitalizations caused by Dengue disease in the state of Paraiba in Brazil.

# Fixed fractals of multivalues maps: <br> MICRO- AND MACRO-FRACTALS 

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The fixed fractal of a multi-valued map $\Phi$ on a topological space $X$ is the closure of the orbit of the set $*_{\Phi}=\{x \in X: x \in \Phi(x)\}$ of fixed points of $\Phi$. For a system $f_{1}, \ldots, f_{n}$ of contracting maps on a complete metric space $X$ the fixed fractal of the multivalued map $\Phi(x)=\left\{f_{1}(x), \ldots, f_{n}(x)\right\}$ coincides with the classical fractal generated by the iterated function system $f_{1}, \ldots, f_{n}$. Given a multi-valued map $\Phi$ on a topological space we shall study the property of its fixed fractal and also the duality between the fixed fractals of the multi-valued map $\Phi$ and its inverse $\Phi^{-1}$. We distinguish two kinds of fixed fractals: micro-fractals and macro-fractals (i.e. fixed fractals for contracting multi-valued maps and their inverses). We shall discuss effective algorithm for drawing fixed fractals and using these algorithm draw macro-fractals which are dual to some well-known micro-fractals (Sierpinski triangle, fractal cross, Koch curve, snowflake, Sierpinski carpet). Finally, we shall discuss the duality between microand macro-fractal dimensions of micro- and macro-fractals.

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# The algebraic Sato-Tate group for abelian VARIETIES 

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Let $l$ be a prime number, $K$ be a number field and let $A$ be an abelian variety over $K$. In an effort of proper setting of the Sato-Tate conjecture concerning the equidistribution of Frobenius elements in the representation of the Galois group $G_{K}$ on the Tate module $T_{l}(A)$, one of attempts is the introduction of the algebraic Sato-Tate group $A S T_{K}(A)$. Maximal compact subgroups of $A S T_{K}(A)(\mathbb{C})$ are expected to be the key tool for the statement of the Sato-Tate conjecture for $A$. At the lecture, following an idea of J-P. Serre (Proc. Sym. in Pure Math. 55), I will present an explicit construction of $A S T_{K}(A)$ based on P. Deligne's motivic category of absolute Hodge cycles (Lect. Notes in Math. 900) and discuss the arithmetic properties of $A S T_{K}(A)$ along with explicit computations of $A S T_{K}(A)$ for families of abelian varieties. This is a joint work with Kiran Kedlaya.

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# On nonlocal First-ORDER EVOLUTION CAUCHY PROBLEMS 

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In the talk, theorems on the existence and uniqueness of mild and classical solutions for some nonlocal first-order evolution Cauchy problems will be discussed.

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# OPTIMAL APPROXIMATION OF STOCHASTIC INTEGRALS <br> With Respect to homogeneous Poisson process OF REGULAR FUNCTIONS IN ASYMPTOTIC SETTING 

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We consider strong approximation of stochastic integrals with respect to a homogeneous Poisson process. An integrand is assumed to be a $r$-smooth function $f:[0, T] \rightarrow \mathbb{R}$. We show that, roughly speaking, the $L^{p}$-error $(p \in[1 ;+\infty))$ of any approximation method, which uses $n$ values of $f$, cannot converge to zero faster than $n^{-r}$ as $n \rightarrow+\infty$. This holds except for a subset of $\mathcal{C}^{r}([0, T])$ with empty interior. The best speed of convergence is achieved by the Itô-Taylor algorithm. This is a joint work with Paweł Przybyłowicz.

# ESTIMATION FOR NON-STATIONARY, LONG MEMORY AND GED-HEAVY TAILS TIME SERIES 

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#### Abstract

Statistical inference for unknown distributions of statistics or estimators based on asymptotic distributions in the case of dependent data is often ineffective. In the last years we can observe a development the of so-called resampling methods. Using these methods, one can directly approximate the unknown distributions of statistics and estimators. A problem that needs to be solved during the study of this procedures is their consistency. In the talk a time series model with specific features: long memory, GEDheavy tails, periodic structure will be consider. Moreover the consistency theorem for the resampling method - subsampling for the estimator of the mean in the considered model will be present.


## References

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# A FEW REMARKS ON CYCLABILITY IN GRAPHS Grzegorz Gancarzewicz <br> Institute of Mathematics <br> Cracow University of Technology <br> Kraków, Poland <br> gancarz@pk.edu.pl 

The set $S$ of vertices is called cyclable in $G$ if all vertices of $S$ belong to a common cycle in $G$. We also speak about cylability or noncyclability of the vertex set $S$.

Several results concerning cyclability will be presented. We are focused on sufficient degree conditions for cyclability of a set $S$ in graphs and bipartite graphs.

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# On Primitive solutions of the Diophantine EQUATION $T^{2}=X_{1}^{5}+X_{2}^{5}+X_{3}^{5}+X_{4}^{5}$ 

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We show that for a broad class of quadratic forms $F$ in three variables, the Diophantine equation $t^{2}=n x y z F(x, y, z)$, where $n \in \mathbb{Z} \backslash\{0\}$, has infinitely many primitive solutions in integers. As an application of our result we prove that for each $n \in \mathbb{Q} \backslash\{0\}$ the Diophantine equation

$$
T^{2}=n\left(X_{1}^{5}+X_{2}^{5}+X_{3}^{5}+X_{4}^{5}\right)
$$

has a solution in co-prime (non-homogenous) polynomials in two variables with integer coefficients. We also present a method which sometimes allow us to prove the existence of primitive integers solutions of more general quintic Diophantine equations of the form $T^{2}=a X_{1}^{5}+$ $b X_{2}^{5}+c X_{3}^{5}+d X_{4}^{5}$, where $a, b, c, d \in \mathbb{Z}$.

## References

[1] M. Gawron, M. Ulas, On primitive integer solutions of the Diophantine equation $t^{2}=G(x, y, z)$ and related results, J. Number Theory vol. 159: 101-122 (2016).

## GAS SpRING FORCE CALCULATOR

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During the lecture we will present the project from Poznan Research Workshop - the algorithm for calculation the force for a gas spring. We will describe a mathematical model of a gas spring and the formula for counting its force. At the end we will show a desktop application, which choses the best fitting gas spring for each input data.

# Key sectors of Polish economy in dynamic Endogenous Input-Output Framework <br> Henryk Gurgul, Łukasz Lach <br> Department of Applications of Mathematics in Economics <br> AGH University of Science and Technology <br> Kraków, Poland <br> henryk.gurgul@gmail.com, llach@zarz.agh.edu.pl 

We propose a new version of dynamic input-output model in which both technological progress and deployment are endogenous, and where sectorspecific outlays on R\&D speed up the development of new technologies and the installation of capital stock. In this two-technology model the new and old technical processes within a sector exchange their relative weights in production. We use the model to obtain projections of the interindustry linkages of sectors in the Polish economy over the next 50 years. The results of this simulation suggest an ongoing change of the composition of the set of key sectors of the Polish economy. In general, one may expect to see an ongoing drop in the importance of agricultureand heavy-industry-related sectors on the one hand, and a rise in the importance of services-related ones on the other.

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## Examples of applications of Jucys-Murphy operators in spin and electron systems

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The set of Jucys-Murphy operators which constitute a complete set of commuting Hermitian operators and generate a maximal Abelian subalgebra in the group algebra $\mathbb{C}\left(\Sigma_{N}\right)$ of the symmetric group $\Sigma_{N}$ is applied in the procedure of an immediate diagonalization of the one-dimensional Heisenberg and Hubbard models. A method for determination of irreducible basis of the Weyl duality based on Jucys-Murphy operators is proposed. The way of construction of appropriate projection operators is pointed out, and the combinatorial meaning of the path on the Young graph, corresponding to a standard Young tableau, is made transparent.

## References

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[2] A. A. Jucys, Factorization of Young projection operators for the symmetric group, Lietuvos Fizikos Rinkinys 11: 5-10 (1971).
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[4] H. Weyl, The Theory of Groups and Quantum Mechanics, Dover, New York, 1950.

# Application of graphs in the construction of Schur-WEYL TRANSFORM 

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The Schur-Weyl transform [1], when applying to spin systems, is a unitary matrix which converts the initial basis of product states with fixed decomposition of spins projections into the irreducible one of the SchurWeyl duality $[2,3]$. We show that such matrix can be constructed by adding the successive spins of atoms according to graph $\Gamma$ with vertices being Gelfand-Tsetlin patterns [4] and edges labelled by single atoms spins projections.

## References

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## GRAPh PRODUCTS AND THEIR PARAMETERS

## Marcin Jurkiewicz

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We present some crucial graph parameters for the strong product of graphs. We have found new results on the edge independence number, i.e. the number of edges in a maximum matching of a graph, and the average distance of the strong product of graphs.

## References

[1] R. Hammack, W. Imrich, S. Klavzar, Handbook of product graphs, CRC Press, Boca Raton, 2010.
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# Simple generalization of median, Bisector AND SYMMEDIAN IN TRIANGLE 

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For any triangle $A B C$ and any integer $n$ we can choose points $D_{n}, E_{n}$, $F_{n}$ on the sides $B C, C A, A B$ respectively, in such a manner that

$$
\frac{|A C|^{n}}{|A B|^{n}}=\frac{\left|C D_{n}\right|}{\left|B D_{n}\right|}, \quad \frac{|A B|^{n}}{|B C|^{n}}=\frac{\left|A E_{n}\right|}{\left|C E_{n}\right|}, \quad \frac{|B C|^{n}}{|A C|^{n}}=\frac{\left|B F_{n}\right|}{\left|A F_{n}\right|} .
$$

Cevians $A D_{n}, B E_{n}, C F_{n}$ are said to be the Maneeals. In the talk we will discuss some properties of the Maneeals and related objects.

## Signature Recognition

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#### Abstract

During the lecture we will present the project from Poznan Research Workshop - the algorithm for signature recognition. We were provided with digital pen, which reads diferent parameters (eg. location, preasure) when signing. Data is presented as a series of points. Our goal was to obtain metric for comparing two signatures, that would allow to setect forgery. The outcome was an application that takes two signatures and returns distance between those signatures as a number. The presented program used Dynamic Time Warping algorithm.


INNER FUNCTION ON ALL SLICES

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We denote $\|f\|_{z}^{2}:=\int_{0}^{1}\left|f\left(e^{2 \pi i t} z\right)\right|^{2} d t$. We assume that $\Omega$ is a circular, bounded, strictly convex domain with $C^{2}$ boundary. Then we construct a nonconstant $f \in \mathcal{O}(\Omega)$ such that $\left\|1-\left|f^{*}\right|\right\|_{z}=0$ for all $z \in \partial \Omega$, where $f^{*}$ denotes radial limit of $f$.

## Approximation of functions by nonlinear SINGULAR INTEGRAL OPERATORS DEPENDING ON TWO PARAMETERS

## Grażyna Krech

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The aim of the talk is to study the behavior of the nonlinear singular integral operators of the form

$$
\begin{equation*}
T_{w}(f)(s)=\int_{G} K_{w}(s-t, f(t)) d t=\int_{G} K_{w}(t, f(s-t)) d t . \tag{1}
\end{equation*}
$$

We investigate the problem of the rate of convergence of the operators (1) at a point $s_{0}$ in which the function $f$ is continuous. This is an extension of the paper by Świderski and Wachnicki ([2]).

This work is a joint project with Eugeniusz Wachnicki.

## References

[1] H. Karsli, Convergence and rate of convergence by nonlinear singular integral operators depending on two parametres, Appl. An. 55: 781-791 (2006).
[2] T. Świderski and E. Wachnicki, Nonlinear singular integrals depending on two parameters, Comment. Math. 40: 181-189 (2000).

# Mathematics - DEDUCTIVE OR EXPERIMENTAL SCIENCE? 

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Most mathematicians spend a lot of time thinking about and analyzing particular examples. This motivates future development of theory and gives one a deeper understanding of existing theory. Gauss declared, and his notebooks attest to it, that his way of arriving at mathematical truths was „through systematic experimentation". It is probably the case that most significant advances in mathematics have arisen from experimentation with examples.

The talk will be about role and history of experiment in mathematics. A few interesting examples will also be presented

## References

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# REMARKS ON COMMUTATIVITY OF RINGS WITH DERIVATIONS 

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Let $R$ be an associative ring (we will usually assume that $R$ is semiprime). In the talk we will discuss some conditions on derivations in $R$ which imply commutativity.

## References

[1] O.D. Artemovych, K. Kular, On the Lie ring of derivations of a semiprime ring, Algebra Discrete Math. 18, no. 2: 157-162 (2014).
[2] K. Kular, Semiprime rings with nilpotent Lie ring of inner derivations, Ann. Univ. Paedagog. Crac. Stud. Math. 13: 103-107 (2014).

# RESAMPLING PROCEDURES FOR NONSTATIONARY TIME SERIES 

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The nonstationary time series play a crucial role in various fields of research such as econometrics, hydrology, mechanical diagnostics etc. In the talk we will present the concept of resampling as an alternative to classical estimation technique. Consistency theorems and practical impact of resampling will be also discussed.

# A test for Bethe Ansatz solution of XXX model for magnetic HEPTAGONAL RING 

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We consider a degenerate doublet within the three-magnon sector at the centre of the Brillouin zone of the heptagonal ring of spins $1 / 2$ within the XXX model. This degeneracy, which originates from parity symmetry, admits an arbitrary choice of basis within this doublet. We point out, however, that Bethe Ansatz solutions impose a definite choice of such a basis, with exactly prescribed spectral parameters. Moreover, the common eigenstates of the complete set of commuting observables emerging from the transfer matrix coincide with the Bethe Ansatz solutions. We demonstrate that rigging of these eigenstates by quasimomenta from the admissible part of the Brillouin zone is fully consistent with both dynamics and parity symmetry.

## References

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# Some properties of the Baskakov-Durrmeyer TYPE OPERATORS OF TWO VARIABLES 

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In this talk we introduce some linear positive operators of the BaskakovDurrmeyer type in the space of continuous functions of two variables. The theorems on convergence and the degree of convergence are established. The Voronovskaya type formula is also presented.

## References

[1] Z. Ditzian, V. Totik, Moduli of Smoothness, Springer-Verlag, New York, 1987.
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# SOME DIOPHANTINE EQUATIONS INVOLVING <br> NUMBERS OF DERANGEMENTS 

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Let $S_{n}$ be the set of permutations of a set with $n$ elements. Recall that the $n$-th number of derangements $D_{n}$ is a number of all permutations in $S_{n}$ without fixed points.

During the talk we will consider diophatnine equation of type

$$
\begin{equation*}
D_{n}=q \cdot m!, \tag{2}
\end{equation*}
$$

where $m, n \in \mathbb{N}_{+}$are unknowns and $q \in \mathbb{Q}_{+}$is a parameter. We will show that for each $q \in \mathbb{Q}_{+}$the equation (2) has only finitely many solutions $(m, n)$. In particular we will give all the indices $n$ for which number $D_{n}$ is a factorial. Moreover, there will be given topological structure (in Euclidean topology) of the set of those parameters $q$ for which the equation (2) has a solution. Finally, we will construct a family of parameters $q$ for which the equation (2) has at least two solutions.

Next we will mention the diophatnine equation

$$
\begin{equation*}
D_{n}=p^{k} \tag{3}
\end{equation*}
$$

where $k, n \in \mathbb{N}_{+}$are unknowns and $p$ is a fixed prime number. We will show that for each $p$ the equation (3) has only finitely many solutions $(k, n)$.

# DEFORMED COHOMOLOGIES OF SYMMETRY PSEUDO-GROUPS AND COVERINGS OF DIFFERENTIAL EQUATIONS 

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I will talk about a relation between deformed cohomologies of symmetry pseudo-groups and coverings of differential equations. Examples will include the potential Khokhlov-Zabolotskaya equation and the BoyerFinley equation.

## References

[1] O.I. Morozov, Deformed cohomologies of symmetry pseudo-groups and coverings of differential equations, arXiv:1509.02716.

## Stabilization problem of Biological population

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The analysis of the stabilization problem of biological population is one of the meaningful questions concern several disciplines including ecology and biology. One of the important issue is how to minimise a range of fluctuation in the population size. Several authors have therefore proposed control strategies to stabilize a population. Recently, some authors have proposed adaptive limiter control (ALC) as a novel method for controlling population oscillations. In the talk, we propose to state the trapping region for metapopulation dynamic and on-population in two regions dynamics. It will presented as additional description models with fractional difference, as an important tool involving some type of memory into system.

## References

[1] D. Franco, and F. M. Hilker, Adaptive limiter control of unimodal population maps, Journal of Theoretical Biology 337: 161-173 (2013).
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# CANONICAL EIGHT-FORMS ON MANIFOLDS <br> With holonomy group $\operatorname{Spin}(9)$ 

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On any 16 -dimensional manifold $M$ with a fixed $\operatorname{Spin}(9)$-structure there exists a canonical 8 -form, which is a unique parallel $\operatorname{Spin}(9)$-invariant 8 -form on $M$ (up to a non-zero factor). In the paper [1] was found an expression $\Omega^{8}=\sum_{i, j, i^{\prime}, j^{\prime}=0, \ldots, 8} \omega_{i j} \wedge \omega_{i j^{\prime}} \wedge \omega_{i^{\prime} j} \wedge \omega_{i^{\prime} j^{\prime}}$ for the canonical 8 -form on $M$ in terms of the $9 \times 9$ skew-symmetric matrix $\omega=\left(\omega_{i j}\right)$ of their local Kähler 2-forms involved. Another expression $\tau_{4}(\omega)=$ $\sum_{0 \leqslant \alpha_{1}<\alpha_{2}<\alpha_{3}<\alpha_{4} \leqslant 8}\left(\omega_{\alpha_{1} \alpha_{2}} \wedge \omega_{\alpha_{3} \alpha_{4}}-\omega_{\alpha_{1} \alpha_{3}} \wedge \omega_{\alpha_{2} \alpha_{4}}+\omega_{\alpha_{1} \alpha_{4}} \wedge \omega_{\alpha_{2} \alpha_{3}}\right)^{2}$ for this 8 -form was proposed in [2]. To prove the non-triviality of $\tau_{4}(\omega)$ in [2] were performed computer computations with the help of the software Mathematica.

We show here that the form $\tau_{4}(\omega)$ is not trivial using the properties of automorphisms of the octonion algebra (without computer computations) and, as a result, that $\Omega^{8}=-4 \tau_{4}(\omega)$.

## References

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[2] M. Parton and P. Piccinni, Spin(9) and almost complex structures on 16-dimensional manifolds, Annals of Global Analysis and Geometry. 41: 321-345 (2012).

Zeta functions, Weil conjectures<br>AND HOW TO APPLY THEM<br>Bartosz Naskręcki<br>Faculty of Mathematics and Computer Science<br>Adam Mickiewicz University<br>Poznań, Poland<br>and<br>Lehrstuhl für Computeralgebra<br>Mathematisches Institut<br>Universität Bayreuth<br>Bayreuth, Germany<br>nasqret@gmail.com

The theory of algebraic varieties over finite fields has grown to a whole separate field of research in the last 50 years. Zeta functions and their properties established in Weil Conjectures proved by Dwork, Grothendieck and Deligne have received much attention. In the talk we will discuss how these cornerstones of arithmetic geometry can be applied to solve problems in the theory of elliptic curves. The point counting technique which lies at the heart of the problem has also applications to coding theory. We will touch upon this problem.

# Assumptions of hypothetical optimization neural NETWORK USING A GENETIC ALGORITHM TO PREDICTIVE APPLICATIONS 

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Optimization method of hypothetical neural network is used mainly to choose the best structures and construction of neural model. Implementation of genetic algorithm to create the structure of the neural network constitutes advanced tool solutions to optimization problems. Optimization process consists of using classical genetic algorithm to maximize the matching function objective (such as frequencies eras).

First stage of optimization is choosing amount of initial solution constituting population genetic neural network. Consecutively in population is carried out section solutions for appropriate task. The best solution is subjected to cross-breeding to a random exchange of parameters between pairs of these solutions. Optimal solutions are submitted to mutation.

## References

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On RINGS WITHOUT *-IDEALS

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In [2] was introduced the concept of $*$-ideals connected with the fundamental property in the theory of radicals of associative rings, namely: for any radical $S$ and any ring $R$, if $I$ is an ideal of $R$, then $S(I)$ is an ideal of $R$. A subring $I$ is called a $*$-ideal of a ring $R$, if $I$ is an ideal of any ring $A$ containing $R$ as an ideal. Obviously, if $S$ is a radical, then $S(R)$ is a a $*$-ideal. Rings without non-trivial $*$-ideals are called $*$-simple. It was proved that a ring is $*$-simple if it is either simple or an algebra with zero multiplication over a field. In [2] was introduced the definition of rings containing no nontrivial some kinds of $*$-ideals. The aim of the talk is to present description of such rings.

## References

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[2] E. R. Puczyłowski, On unequivocal rings, Acta Math. Hungar., 36 vol. 1: 57-62 (1980).

# Richness of The family of Bounded operators ON A BANACH SPACE 

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#### Abstract

W.T. Gowers proposed in 2002 the programme of loose classification of Banach spaces with respect to richness of the family of bounded operators inside the space. We sketch development of the programme and present some recent results concerning Gowers' classification programme from the joint work with S.A. Argyros and A. Manoussakis.


## References

[1] S.A. Argyros, A. Manoussakis and A. Pelczar-Barwacz, Quasiminimality and tightness by range in spaces with unconditional basis, Israel J. Math. 200 (1): 19-38 (2014).
[2] A. Manoussakis and A. Pelczar-Barwacz, Types of tightness in spaces with unconditional basis, Studia Math. 220 (3): 243-264 (2014).
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## A CERTAIN ESTIMATE FOR POLYNOMIALS

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One of the most important polynomial inequalities is the following Markov's inequality.

Theorem (Markov, 1889) If $P$ is a polynomial of one variable, then

$$
\left\|P^{\prime}\right\|_{[-1,1]} \leq(\operatorname{deg} P)^{2}\|P\|_{[-1,1]}
$$

Moreover, this inequality is optimal, because for the Chebyshev polynomials $T_{n}\left(n \in \mathbb{N}_{0}\right)$, we have $T_{n}^{\prime}(1)=n^{2}$ and $\left\|T_{n}\right\|_{[-1,1]}=1$.

Recall that

$$
T_{n}(u)=\frac{1}{2}\left[\left(u+\sqrt{u^{2}-1}\right)^{n}+\left(u-\sqrt{u^{2}-1}\right)^{n}\right] .
$$

From the point of view of applications, it is important that the constant (deg $P)^{2}$ in Markov's inequality grows not too fast (that is, polynomially) with respect to the degree of the polynomial $P$.

It is natural to ask about similar inequalities if we replace the interval $[-1,1]$ by another compact set in $\mathbb{R}^{N}$ or $\mathbb{C}^{N}$. In the talk, we will address this issue.

# SOME CLASSICAL LOWER BOUNDS ON THE <br> INDEPENDENCE NUMBER AND THEIR BEHAVIOR FOR THE STRONG PRODUCT OF GRAPHS 

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We present the Shannon's model of a noisy communication channel introduced in 1956. In the first part, we quote the zero-error capacity of the mentioned channel and we show the Shannon's method to calculate it directly. In the second part, we analyze some classical lower bounds on the independence number of a graph in the context of the Shannon capacity, e.g. Caro-Wei and Aouchiche-Brinkmann-Hansen bounds.

## References

[1] A. Brouwer, W. Haemers, Spectra of graphs, Springer, Amsterdam, 2010.
[2] C. Shannon, The zerro error capacity of a noisy channel, IEEE Trans. Inform. Theory IT-2: 8-19 (1956).

# On REGULARIZATION OF PLURISUBHARMONIC FUNCTIONS NEAR BOUNDARY POINTS 

## Szymon Pliś

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We prove in an elementary way that for a Lipschitz domain $D \subset \mathbb{C}^{n}$, all plurisubharmonic functions on $D$ can be regularized near any boundary point.

## References

[1] S. Pliś, On regularization of plurisubharmonic functions near boundary points, arXiv:1412.0562

# Incremental Set Cover Problem 

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During the lecture I will present the project - Incremental Set Cover realized on Research in Industrial Projects for Students at University of California in Los Angeles.
Efficient retrieval of information is of key importance when using Big Data systems. In large scale-out data architectures, data are distributed and replicated across several machines. Queries/tasks to such data architectures, are sent to a router which determines the machines containing the requested data. Ideally, to reduce the overall cost of analytics, the smallest set of machines required to satisfy the query should be returned by the router. Mathematically, this can be modeled as the set cover problem, which is NP-hard, thus making the routing process a balance between optimality and performance. Even though an efficient greedy approximation algorithm for routing a single query exists, there is currently no better method for processing multiple queries than running the set cover algorithm repeatedly for each query. This method is impractical for Big Data systems and the state-of-the-art techniques route a query to all machines and choose as a cover the machines that respond fastest. In this talk, I will show an efficient technique to speedup the routing of large number of real-time queries while minimizing the number of machines that each query touches (query span). Experiments show that our incremental set cover-based routing is 2.5 A - faster and can return on average $50 \%$ fewer machines per query when compared to repeated greedy set cover and baseline routing techniques.

# A HYBRID MODEL OF TUMOUR GROWTH AND ANGIOGENESIS 

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The aim of this study was to develop the model of tumour growth and angiogenesis in three-dimensional space. Angiogenesis is a process of growth of new blood vessels from existing vasculature. It often takes place in case of invasive tumour growth when cells are under to the hypoxic conditions. In order to model tumour the multiphase theory was applied. Three types of cells are distinguished: proliferating, quiescent and necrotic. Continuous part of the model is complemented by reactiondiffusion equations in order to model changes in concentrations of oxygen and vascular endothelial growth factor within the tissue. The process of angiogenesis is modeled by a discrete model. It enables modelling growth and sprouting of the new vessels, creation of functional vessel loops and simulation of the blood flow. A number of simulations were performed with different parameter settings. In enabled analysis of structure of the vasculature, overall tumour volume as well as changes of hematocrit and blood flow. Developed model provides framework for further investigations including modelling of 3D distribution of the drugs and optimal drug treatment. This work was supported by the National Science Centre (NCN) under grant 2011/03/B/ST6/04384, and performed using the infrastructure supported by POIG.02.03.01-24-099/13 grant: GCONiI -Upper-Silesian Center for Scientific Computation.

# A FEW REMARKS ON THE CHARACTERISTIC MAP and Helton-Rosenthal-Wang theorem 

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Let $\mathcal{M}_{n}(\mathbb{F})$ be the space of all $n \times n$ matrices over a field $\mathbb{F}$. For $k \in$ $\{1, \ldots, n\}$ define $\mathrm{S}_{k}(A)$ to be the sum of all $k \times k$ principal minors of a matrix $A \in \mathcal{M}_{n}(\mathbb{F})$. In the talk, some properties of the characteristic map

$$
\chi: \mathcal{M}_{n}(\mathbb{F}) \ni A \longmapsto\left(\mathrm{~S}_{1}(A), \ldots, \mathrm{S}_{n}(A)\right) \in \mathbb{F}^{n}
$$

will be discussed. We will focus on the Helton-Rosenthal-Wang theorem on images under $\chi$ of affine subspaces.

## References

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# Estimating The power parameter in Generalized GaUssian distribution: STUDY of difficulties and NUMERICAL EXPERIMENTS 

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We consider a challenging problem of estimating the shape (power) parameter $\alpha$ in symmetric, zero-mean, unit-variance Generalized Gaussian class, also known as Generalized Error Distribution. This parameter is crucial in modelling heavy tails, eg. in financial time series returns, signal processing. Moment estimators are considered in the context of their possible non-existence for small and moderate sample sizes. Even though non-parametric bootstrap for the parameter involved does not substantially improve the estimator performance, we prove the consistency of the method. We also address the Maximum Likelihood estimation, which provides more reliable results. Extensive computer simulation study (Monte Carlo and bootstrap), including real data analysis, is provided for illustrative purposes.

## References

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# Subnormality of weighted shifts on directed TREES AND COMPOSITION OPERATORS IN $L^{2}$-SPACES 

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A question of subnormality of composition operators in $L^{2}$-spaces over possibly simplest (excluding the case of classical weighted shifts) discrete measure spaces is discussed. We restrict ourselves to the case of composition operators built over connected directed graphs whose vertices, all but one, have valency one. This includes the class of weighted shifts on directed trees with one branching vertex and with infinite trunk, as well as the class of composition operators over the directed graph with one branching vertex, a circuit of length $\kappa+1$ and $\eta$ branches. The former class has been intensively studied since 2012. The latter class is new. It has unexpected properties. In particular, we will show that there exists a nonhyponormal composition operator in the $L^{2}$-space built over a directed graph with one loop $(\kappa=0)$ and two branches $(\eta=2)$, which generates Stieltjes moment sequences. Overview of previous results concerning weighted shifts on directed trees will be presented as well.

## References

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# Selections of set-valued functions <br> SATISFYING A LINEAR INCLUSION 

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Let $(Y,\|\cdot\|)$ be a real Banach space and $K$ a convex cone in a vector space $X$. Consider a set-valued function $F: K \rightarrow c l(Y)$ satisfying the following inclusion

$$
\alpha F(x)+\beta F(y) \subset F(p x+q y), \quad x, y \in K
$$

We determine conditions for which the s.v. function $F$ admits a selection satisfying the corresponding functional equation. We also adopt the method of the proof for investigating the stability of general linear equation.

The talk is based on a joint work with Andrzej Smajdor.

## References

[1] A. Smajdor and J. Szczawińska, Selections of set-valued functions satisfying the general linear inclusion, J. Fixed Point Theory Appl., DOI 10.1007/s11784-015-0265-9.
[2] A. Smajdor and J. Szczawińska, Selections of generalized convex setvalued functions with bounded diameter, Fixed Point Theory (to appear).

## RECENT WORK ON IMPORTANT THEOREMS THAT FAIL IN SOME MODELS FOR ZF

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It will be proved that the following statements are all consistent with ZF:
(i) There exists an uncountable set $J$ such that $\mathbb{R}^{J}$ is metrizable.
(ii) There exists an uncountable set $J$ such that the Cantor cube $2^{J}$ is first-countable.
(iii) A Hausdorff compact space which contains a dense completely regular subspace need not be completely regular.
(iv) For non-equivalent Hausdorff compactifications $\alpha X$ and $\gamma X$ of an infinite discrete space $X$ and for each real function $f$ on $X$, it may happen that $f$ is continuously extendable over $\alpha X$ if and only if $f$ is continuously extendable over $\gamma X$.

It holds true in $\mathbf{Z F}$ that statements (i) and (ii) are equivalent with the negation of CC(fin).

## References

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[3] H. Herrlich, Axiom of Choice, Springer-Verlag, Berlin, 2006.

# Log-Periodic Power Law and Generalized Hurst Exponent analysis in Estimating An Asset bubble BURSTING TIME 

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Speculative bubbles and the subsequent crashes are an integral part of human history. Greed and fear present in the financial markets naturally favor the occurrence of extreme events. Despite the research conducted for a long time in this topic, there is still no clear consensus on the definition and the causes of speculative bubbles. Their correct identification and forecasting in advance are still unresolved problems.

In my presentation I will discuss two practical models: Log-Periodic Power Law and Generalized Hurst Exponent. They were tested on 10 historical bubbles and then applied to the current situation on financial markets. I will describe my methods and show the most important results. Especially interesting was detection of peaks on German and Chinese stock markets in 2015 which was achieved in advance.

# Industrial Mathematics - What is it <br> AND HOW TO DO IT? 

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Industrial Mathematics is a branch of mathematics in which problems are solved by mathematicians and reported by widely understood the industry or business. During the lecture we will talk about the activities of the industrial mathematics in the context of Poznan research centers. The most important event is the Poznań Research Workshop (PPB) - based on the best of British experience of workshops to promote cooperation between the scientific community and the businesses and institutions. The goal of the workshop is to solve problems in the field of industrial mathematics, and creating new or improving: processes, products and services in companies and institutions with the use of mathematical tools and computer science. Similar events are organized successfully all around the world for decades and attract the best students, phDs and professors from the best universities. During the presentation it will be given an examples of projects and what it is in practice industrial math.

## Nonparametric Confidence Bands in Wicksell's Problem

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Consider a population of spheres of random radii, randomly distributed in some three-dimensional opaque medium. The goal is to estimate the density of those spheres radii, when data are available only from random plane slices through the medium. This problem, first posed and theoretically solved by Wicksell (1925), is a classical problem in stereology, which arises, e.g., in biology, astronomy, geology, metallurgy and many others.

In this talk, a kernel-type nonparametric estimator for the density of interest will be constructed along with corresponding asymptotic uniform confidence bands. The performance of the procedures will be investigated in a simulation experiment and demonstrated with some real astronomical data related to M62 globular cluster.

## References

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# On some cancellation algorithms 

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Let $f$ be a natural-valued function defined on the Cartesian product of finitely many copies of $\mathbb{N}$. Here we will discuss some modifications of the sieve of Eratosthenes in the sense that we cancel the divisors of all possible values of $f$ in the points whose sum of coordinates is less or equal to $n$. By applying similar arguments to those used in the paper of Browkin and Cao[1], but also in the companion papers ([2] and [3]), we investigate new problems for the values of some polynomial functions or quadratic and cubic forms.

## References

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