Workshop on Modern Applied Mathematics PK 2016

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Book of Abstracts

Workshop on Modern Applied Mathematics
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dr hab. Anna Pelczar-Barwacz (Jagiellonian University)
prof. dr hab. Zbigniew Szafraniec (University of Gdańsk)

Contents

Introduction

Workshop on Modern Applied Mathematics PK 2016 is the fifth edition of an annual conference on modern mathematics organized by the Institute of Mathematics of the Faculty of Physics, Mathematics and Computer Science, Tadeusz Kościuszko Cracow University of Technology.

The Conference aims to present new results, to promote and to bring together researchers in the different research areas of mathematics and influence more cooperation among scientists working in mathematics. This conference will provide a unique forum for exchanging ideas and in-depth discussions on different aspects and different branches of mathematics.

You can find detailed information about conference on the site:

`www.wmam.pk.edu.pl`

After the conference a special issue of Technical Transactions will be published and every participant is invited to submit an article.

I would like to thank professors Tadeusz Krasiński, Marek Kubale, Krzysztof Maślanka, Witold Mozgawa, Wiesław Pawłucki, Anna Pelczar-Barwacz and Zbigniew Szafraniec for accepting invitation to give a lecture during the conference and all participants for interest in the fourth edition of our conference and scientific research in the field of mathematics.

The conference is under the media patronage of Welcome Cracow.

I would like to express my thanks to Board of Directors and the Administration of the Institute of Mathematics as well as the staff of the Institute for their friendliness and support for conference organization.

On the behalf of the organizing committee
Grzegorz Gancarzewicz

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Abstracts

Approximation by positive linear operators

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From the well-known Korovkin theorem, we have the convergence $L_n(f) \to f$ in the uniform norm for all $f \in C[0,1]$, if it holds for the test functions $e_i(x) = x^i$, $i = 0, 1, 2$ (see [3]). We will discuss the Korovkin-type theorems and the rate of convergence of positive linear operators.

References

[1] F. Altomare, Korovkin-type theorems and approximation by positive linear operators, *Surv. Approx. Theory* **5**: 92–164 (2010).

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[3] P. P. Korovkin, Convergence of linear positive operators in the spaces of continuous functions (Russian), *Doklady Akad. Nauk. SSSR (N.S.)* **90**: 961–964 (1953).

[4] O. Shisha and B. Mond, The degree of convergence of sequences of linear positive operators, *Proc. Nat. Acad. Sciences U.S.A.* **60**: 1196–1200 (1968).

The Hodge numbers of Calabi–Yau threefolds of Borcea–Voisin type

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Calabi–Yau treefold obtained as a crepant resolution of variety $S \times E/(\alpha_S \times \alpha_E)$, where $\alpha_S \in \mathrm{Aut}(S)$ and $\alpha_E \in \mathrm{Aut}(E)$ are purely nonsymplectic automorphisms is called Borcea–Voisin type. In [1] generalized Borcea–Voisin Calabi–Yau treefolds were studied for automorphisms of order 2,3,4 and 6. We shall verify formulas for the Hodge numbers using orbifold formulas introduced by Chen and Ruan in [2].

References

[1] A. Cattaneo, A. Garbagnati, Calabi-Yau 3-folds of Borcea–Voisin type and elliptic fibrations, arxiv.org/abs/1312.3481.

[2] W. Chen, Y. Ruan, A new cohomology theory of orbifold, *Comm. Math. Phys.* **248**, No. 1: 1–31 (2004).

Automatic sequences and nilmanifolds

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We conjecture that generalised polynomial functions cannot be generated by finite automata, except for the trivial case when they are ultimately periodic. Using methods from ergodic theory, we are able to partially resolve this conjecture, proving that any hypothetical counterexample is periodic away from a very sparse and structured set. To this end, we use the results of Bergelson-Leibman that relate generalised polynomials to dynamics on nilmanifolds.

The results obtained lead to a number of interesting questions concerning sparse zero sets of generalised polynomials that are of independent interest.

The talk is based on joint work with Jakub Konieczny.

References

[1] V. Bergelson and A. Leibman, Distribution of values of bounded generalized polynomials, *Acta Math.*, **198**, No. 2: 155–230 (2007).

[2] J. Byszewski and J. Konieczny, Automatic sequences, generalised polynomials, and nilmanifolds, arxiv.org/abs/1610.03900.

Some remarks on nonlocal initial conditions

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In the talk, some remarks on nonlocal initial conditions, in the sense of [2] and [1], will be discussed.

References

[1] L. Byszewski, *Wybrane zagadnienia z równań i nierówności różniczkowych i funkcjonalno-różniczkowych z warunkami nielokalnymi*, Monografia 505, Politechnika Krakowska, Kraków 2015.

[2] J. Chabrowski, On non-local problems for parabolic equations, *Nagoya Math. J.* **93**: 109–131 (1984).

Regulous functions

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The aim of a talk will be to briefly present the ring of rational functions admitting a continuous extension to the real affine space (including examples). I would like to establish some interesting properties of this ring - in particular Nullstelensatz. Finally, an unpublished new fact about ring of regulous functions will also be announced.

References

[1] G. Fichou, J. Huisman, F. Mangolte, J.-P. Monnier, Fonctions regulues, arxiv.org/abs/1112.3800.

[2] G. Fichou, J.-P. Monnier, R. Quarez, Continuous functions in the plane regular after one blowing-up, arxiv.org/abs/1409.8223.

[3] J. Kollar, K. Nowak, Continuous rational functions on real and p-adic varieties II, arxiv.org/abs/1301.5048.

The optimal block length for the subsampling in specific case of time series (long memory, heavy tails)

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Statistical inference for unknown distributions of statistics or estimators may be based on asymptotic distributions. Unfortunately, in the case of dependent data the structure of such statistical procedures is often ineffective. Using so-called resampling methods, it is possible to directly approximate the unknown distributions of statistics and estimators.
Important issue, that needs to be solved during the study of the resampling procedures is choosing the optimal block length for this techniques. In the talk this problem, for time series with specific features i.e.: long memory, heavy tails and periodic structure, will be discussed and the simulation study will be presented.

References

[1] P. Bertail, Comments on: Subsampling weakly dependent time series and application to extremes, *TEST* **20**: 487-490 (2011).

[2] E. Gajecka, Subsampling for weakly dependent and periodically correlated sequences, in: *Cyclostationarity: Theory and Methods*, Lecture Notes in Mechanical Engineering, Springer, 2014.

A DEGREE SUM CONDITION ON HAMILTONIAN CYCLE THROUGH SPECIFIED EDGES IN BIPARTITE GRAPHS

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This is a joint work with Denise Amar, Evelyne Flandrin, A. Paweł Wojda. For a bipartite graph G we present degree sum conditions for couples of independent vertices from different partite sets under which the graph G is hamiltonian and we give their analogs for hamiltonian cycles through specified sets of independent edges.

Approximation of functions in Orlicz spaces

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The aim of the talk is to detail a fragment of the scientific work of Professor E. Wachnicki. We shall recall theorems which were proved by Professor E. Wachnicki in the year 1976. The idea of a new approach to approximation problems in Orlicz spaces will be presented.

References

[1] M. A. Krosnosel'skii, Ya. B. Rutickii, *Convex functions and Orlicz spaces*, P. Noordfoff Ltd., Groningen, 1961 (translated by L. F. Boron).

[2] E. Wachnicki, Approximation of functions in Orlicz space, *Comment. Math.* **19**: 161–164 (1976).

[3] W. Rudin, *Real and complex analysis*, McGraw-Hill, 1987.

A rational counterexample to a certain containment

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The talk will be connected with the special case of the problem of containment relation between symbolic and ordinary powers of homogeneous ideals. In 2001 it was proved that containtment

$$I^{(m)} \subset I^r$$

holds, if m is greater than r multiplied by the dimension of the ambient space. This result was expected to be improved. One of possible improvements would be

$$I^{(3)} \subset I^2.$$

The first counterexample was given over the complex numbers by Dumnicki, Szemberg and Tutaj-Gasińska. I will report on the recent paper in which Lampa-Baczyńska and Szpond described the first rational counterexample. This is also the only known up to now rational counterexample.

References

[1] M. Lampa-Baczyńska, J. Szpond, From Pappus Theorem to parameter spaces of some extremal line point configurations and applications, to appear in *Geom. Dedicata* (arxiv.org/abs/1509.03883).

Functional data clustering

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This talk will present cluster analysis with respect to the functional data. The approaches applied for clustering functional data are based on the methodologies such as dimension reduction before clustering and model-based clustering methods. The presentation consists of four parts. In the first part the basic FDA concepts will be mentioned. Afterwards, the main definitions and theorems in cluster analysis will be shown. In the third part the functional data clustering concepts will be introduced. Last part is practical and example of clustering data analysis will be presented.

References

[1] T. Hastie, R. Tibshirani, J. Friedman, *The Elements of Statistical Learning: Data Mining, Inference and Prediction*, Springer, 2008.

[2] L. Horváth, P. Kokoszka, *Inference for Functional Data with Applications*, Springer, 2012.

[3] J. Jacques, C. Preda, Functional data clustering: a survey, *Advances in Data Analysis and Classification* **8**, No. 3: 231–255 (2014).

Hausdorff limits of one parameter families of definable sets in o–minimal structures

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The talk will be about an elementary proof of the following theorem on definability of Hausdorff limits of one parameter families of definable sets: let $A \subset \mathbb{R} \times \mathbb{R}^n$ be a bounded definable subset in o–minimal structure on $(\mathbb{R}, +, \cdot)$ such that for any $y \in (0, c), c > 0$, the fiber $A_y := \{x \in \mathbb{R}^n : (y, x) \in A\}$ is a Lipschitz cell with constant L independent of y. Then the Hausdorff limit $\lim\limits_{y \mapsto 0} \bar{A}_y$ exists and is definable.

References

[1] L. Bröcker, Families of semialgebraic sets and limits, in: *Real algebraic geometry, Proc. Conf., Rennes/Fr. 1991*, Lect. Notes Math. 1524: 145–162 (1992).

[2] L. van den Dries, *Tame topology and o-minimal structures*, London Mathematical Society Lecture Note Series 248, Cambridge University Press, 1998.

[3] B. Kocel-Cynk, W. Pawłucki, A. Valette, A short geometric proof that Hausdorff limits are definable in any o-minimal structure, *Adv. Geom.* **14**, No. 1: 49–58 (2014).

Periodic points of polynomials and algebraic units

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Let D be an integral domain and K the algebraic closure of the fraction field of D. In the talk, a certain theorem, concerning a quite efficient method of producing invertible elements of the integral closure of D in the field K, will be discussed. The theorem is a consequence of a surprising identity which involves periodic points of a polynomial. The talk will be based on Robert L. Benedetto's paper "An elementary product identity in polynomial dynamics" (*Am. Math. Mon.* **108**, No. 9: 860–864 (2001)).

Copula functions in modelling financial time series

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The theory of copula functions allows modeling of multivariate distributions using one–dimensional marginal distributions and certain copula function. The main purpose of the talk is to present copula functions and discuss the method of selection of copulas for two–dimensional investment portfolio based on necessity to reject the normality of presented components.

References

[1] U. Cherubini, E. Luciano and W. Vecchiato, *Copula methods in finance*, John Wiley & Sons, 2004.

[2] C. Francq, J. M. Zakoian, *GARCH models: structure, statistical inference and financial applications*, John Wiley & Sons, 2011.

[3] J. Leśkow, J. Mokrzycka, K. Krawiec, Modeling stock market indexes with copula functions, *Finansowy Kwartalnik Internetowy e–Finanse* **2**: 1–16 (2011).

Estimation with missing data

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The presentation consist of few parts. First of them is short introduction to missing data. Types of missing data and reasons of missingness will be presented. In the next part we will focus on methods of estimation for incomplete data. Likelihood, imputation and inverse probability weighted estimators will be shortly presented. In the last part, double robust estimator will be discussed.

References

[1] A. Tsiatis, *Semiparametric Theory and Missing Data*, Springer, 2006.

[2] J. Kang, J. Schafer, Demystifying Double Robustness: A Comparison of Alternative Strategies for Estimating a Population Mean from Incomplete Data, *Stat. Sci.* **22**, No. 4: 523–539 (2007).

[3] H. Bang, J. Robins, Doubly Robust Estimation in Missing Data and Casual Inference Models, *Biometrics* **61**: 962–972 (2005).

Approximation by some exponential type operators

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We consider an operator S_λ ($\lambda \in \mathbb{R}$) defined by

$$S_\lambda(f,t) = \int_A^B W(\lambda,t,u)f(u)\,du, \quad -\infty \le A < B \le +\infty,$$

where a kernel W is a positive function, satisfying the normalization condition $\int_A^B W(\lambda,t,u)du = 1$ and the following equation

$$\frac{\partial}{\partial t}W(\lambda,t,u) = \frac{\lambda(u-t)}{p(t)}W(\lambda,t,u) - \beta W(\lambda,t,u)$$

with $\beta \ge 0$, $\lambda \in \mathbb{R}$, $u,t \in (A,B)$, a function p analitic and positive for $t \in (A,B)$. It is similar to a family of operators considered by Ismail and May and all these operators are approximation. Moreover, a rate of convergence, the Voronovskaya type formula and some limit problems will be presented. The talk is based on joint work with Eugeniusz Wachnicki.

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Jumps of Milnor numbers in families of singularities

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Let f_0 be an isolated singularity at $0 \in \mathbb{C}^n$, i.e. f_0 is a holomorphic function in a neighbourhood of $0 \in \mathbb{C}^n$ such that

1. $f_0\,(0) = 0$,
2. $\nabla f_0\,(0) := (\frac{\partial f_0}{\partial z_1}(0), \ldots, \frac{\partial f_0}{\partial z_n}(0)) = 0$,
3. $\nabla f_0\,(z) \neq 0$ for $z \neq 0$ sufficiently small.

The basic topological invariant of the singularity f_0 is the Milnor number $\mu(f_0) \in \mathbb{N}$ defined as the index at $0 \in \mathbb{C}^n$ of the vector field generated by the gradient ∇f_0 of f_0. In the lecture I will describe changes of the Milnor number in holomorphic families of singularities.

On some Gauss-Weierstrass generalized integrals

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The aim of this talk is to present approximation properties of the operator W_α defined by

$$W_\alpha(f)(r,t) = W_\alpha(f;r,t) = \int_0^\infty K_\alpha(r,s,t)f(s)\,ds,$$

where $\alpha \geq -\frac{1}{2}$, $r>0$, $t>0$, $K_\alpha(r,s,t) = \frac{1}{4t}\left(\frac{s}{r}\right)^{\frac{\alpha}{2}} \exp\left(-\frac{r+s}{4t}\right) I_\alpha\left(\frac{\sqrt{rs}}{2t}\right)$, and I_α is a modified Bessel function

$$I_\alpha(z) = \sum_{k=0}^\infty \frac{z^{\alpha+2k}}{2^{\alpha+2k}k!\Gamma(\alpha+k+1)}.$$

In particular, a rate of convergence, the Voronovskaya type formula and some limit problems will be presented.

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Small graphs can cause big coloring problems

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On the whole, the performance of graph coloring heuristics is studied by giving asymptotic results. These are usually the performance guarantee and time complexity. Both functions tell us what one can expect at worst as the number of vertices n tends to infinity, but we really do not know what is going on at the other end of the scale, say if n<10. For this reason Hansen and Kuplinsky [2] introduced the concept of hard-to-color (HC) graphs. These are graphs that cannot be colored optimally by some approximation algorithms. The aim of studying such graphs is threefold. First, analyzing HC graphs makes it possible to obtain improved algorithms which avoid hard instances as far as possible. Secondly, it enables to search for small benchmarks, which is an indispensable tool in evaluating comprehensive families of graph coloring heuristics. Thirdly, it proves a more sensitive way of assessing their efficiency as compared to the performance guarantee (the larger HC graph is the better algorithm performs), since the overwhelming majority of coloring algorithms have asymptotically the same linear function of performance guarantee. In the talk we review the most popular on-line and off-line graph coloring algorithms. For each algorithm we give: a short description, performance guarantee, the smallest HC and slightly HC graphs, positive cases (i.e. optimally colorable graphs) and negative cases. Finally, we give the smallest benchmark for off-line sequential algorithms and the smallest weak benchmark for on-line algorithms.

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Remarks on δ-(semi)prime rings and δ-nilpotent elements

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Let δ be a derivation on an associative ring R (possibly without identity). In the talk, we will provide a brief introduction to the notion of a δ-nilpotent element in R. We will also discuss relationships between the δ-nilpotent elements and the δ-semiprimeness of R.

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Modelling discrete time series from the bootstrap perspective

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The talk will be dedicated to modelling time series with discrete values. The usual ARMA models fail due to the value constraints. A class of generalized ARMA models (GARMA models) will be therefore considered and estimation properties discussed. Application of bootstrap technique will be show. Such models will be show to be useful in analyzing and predicting incidences of dengue disease in Brazil.

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Approximation of functions of several variables by the Baskakov-Durrmeyer type operators

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In this paper we introduce some linear positive operators of the Baskakov-Durrmeyer type in the space of continuous functions of several variables. The theorem on the degree of convergence is established. Moreover, we give the Voronovskaya type formula for these operators.

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Number theory and experimental mathematics

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Computers were designed and constructed by several eminent mathematicians: John von Neumann, Alan Turing, Howard Aiken, John Atanasoff. No one can deny prominent role of computers in purely numerical engineering calculations. However, in recent years we have been witnessing quite new phenomenon known as computer assisted proofsorcomputer aided proofs. Even more, quite new and fascinating branch of science have emerged: experimental mathematics. The role of computers in modern mathematics may be compared to that of a telescope in astronomy or a microscope in biology. With the help of both devices we can reveal and visualize some restricted fragment of reality. Nevertheless, none these devicescan provide us with a sufficient deepexplanation of a given fact. In other words, none of them can provide us with a complete andrigorous proof which always have been and will always be the corner stone of all mathematics. In this talk I will give some intriguing examples of computer experiments in one of the oldest and most distinguished branches of mathematics: number theory.

On some generalisation of evolution equation with non-local initial conditions

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We consider the possibility of generalisation of Theorem of Byszewski, concerning semilinear evolution equations with non-local initial conditions on the case of stochastic Ito equation.

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A note on p-adic valuations of Stirling numbers of the second kind

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Let $n, k \in \mathbb{N}_+$. The Stirling number of the second kind $S(n,k)$ counts the number of partitions of a set with n elements into exactly k nonempty subsets. The exact formula for Stirling numbers of the second kind is following

$$S(n,k) = \frac{1}{k!}\sum_{j=1}^{k}(-1)^{k-j}\binom{k}{j}j^n.$$

The problem of computation of p-adic valuations (with emphasize on 2-adic valuations) of Stirling numbers of the second kind and their relatives generated a lot of research, e.g. Lengyel (1994 and 2009), Clarke (1995), Gessel and Lengyel (2001), De Wannemacker (2005), Hong, Zhao and Zhao (2012), Bennet and Mosteig (2013). Amdeberhan, Manna and Moll in 2008 and Berrizbeitia, Medina, Moll, Moll and Noble in 2010 stated two conjectures on behavior of p-adic valuations of numbers $S(n,k)$.

The aim of this talk is to establish the mentioned conjectures. The proof is based on elementary facts from p-adic analysis.

Ovals and isoptics

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A set of points from which an oval C is seen under the fixed angle $\pi - \alpha$ is said to be an α-isoptic of the oval C. Isoptics were considered by P. de La Hire and M. Chasles. Nowadays J. W. Green in [7], S. Matsuura in [8] also W. Wunderlich in [17] as well as in [18], described curves for which their certain isoptic curve is either a circle or an ellipse. W. Wunderlich in [19], gave some applications of isoptics to engineering in cam mechanisms theory. Recently G. Csima and J. Szirmai in [5] and [6] try to extend the theory of isoptics to higher dimensions.

In this lecture we will introduce a wide range of properties of isoptics of ovals and strictly convex plane closed curves.

We begin with the theorems on convexity of isoptics ([2], [3],[9]), various Cauchy-Crofton-type formulas ([2], [3], [10], [11], [14]), a generalization of the notion of a curve of constant width and a generalization of Mellish theorem on constant width curves ([12], [15]).

Next, we consider the family of regular plane closed strictly convex curves and the family of differential equations of special type associated to positive-valued, periodic and continuous functions. Each curve and the solutions of a fixed differential equation determine a family of disjoint closed curves called φ-optics. In particular cases we obtain the family of α-isoptics and the family of arc-optic curves. The family of arc-optic curves is closely related to the solutions of the Ulam floating body problem. We generalize the Bianchi-Auerbach equation derived for the problem of floating body to general case of φ-optics and we give some its applications. We present the necessary and sufficient conditions for some geometric quantities connected in a natural way to φ-optics to be constant ([4]).

In a subsequent part of the lecture we introduce notion of the isoptics of pairs of nested closed strictly convex curves and present some their geometric properties and Crofton-type formulas ([10]).

We pass next to Poncelet's porism and bar billiards, and we give the theorem that for a given oval C there exist ovals C_{in} and C_{out}, inside and outside of C, such that the pairs (C, C_{in}) and (C_{out}, C) satisfy the Poncelet's porism for almost any number of reflections in their bar billiards.

In these considerations we introduce and use a certain generalization of the isoptic. The two tangent lines determining a point ona the isoptic determine also a secant of the oval, and the envelope of all these secants is called an inner isoptic of these curves ([13]).

Next topic is to describe a certain integral formula for inner isoptics in terms of quantities that naturally occur in this geometric configuration ([11]).

We pass now to two facts on the relations of a rotation index of the bar billiards of two nonconcentric nested circles with a Poncelet's prism property. The rational indexes correspond to closed Poncelet's transverses without or with self-intersections. We give a certain interesting series arising from the theory of special functions and relating the rotation number equal to $\frac{1}{3}$ given by a triangle formed by Poncelet's transverses with a double series involving the radii of the circles and the offset of their centers. The second fact provides a Steiner-type formula which gives a necessary condition for a bar billiard to be a pentagon with self-intersections for which rotation index equal to $\frac{2}{5}$, ([1]).

At the end we provide the secantoptics – an another generalization of the isoptics. We show that any evolutoid of a given oval is a hedgehog and that any secantoptic of an oval is an isoptic of a pair of certain evolutoids. We prove some Crofton-type formulas for secantoptics and give a necessary and sufficient condition for a secantoptic to be convex ([16]).

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Reduction of invariant bi-Poisson structures and Dirac brackets

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Let X be a manifold with a bi-Poisson structure generated by a pair of G-invariant symplectic structures ω_1 and ω_2, where the Lie group G acts properly on X. Let H be some isotropy subgroup for this action representing the principle orbit type and $X_{\mathfrak{h}}$ be the submanifold of X consisting of the points in X with the stabilizer algebra equal to the Lie algebra $\mathfrak{h}$ of H and with the stabilizer group conjugated to H in G. We prove that the pair of symplectic structures $\omega_1|_{X_{\mathfrak{h}}}$ and $\omega_2|_{X_{\mathfrak{h}}}$ (Dirac brackets) generates an $N(H^0)/H^0$-invariant bi-Poisson structure on $X_{\mathfrak{h}}$, where $N(H^0)$ is the normalizer in G of the identity component H^0 of H. The action of $N(H^0)/H^0$ on $X_{\mathfrak{h}}$ is locally free and proper. Since the proper action of the group $N(H^0)/H^0$ on the manifold $X_{\mathfrak{h}}$ is locally free, for investigation of the bi-Poisson algebra of G-invariant functions on X (isomorphic to the bi-Poisson algebra of $N(H^0)/H^0$-invariant functions on $X_{\mathfrak{h}}$) we can use well known methods developed for locally free actions. For example, applying these methods based on the moment map theory, we can investigate the Kronecker property of the bi-Poisson algebra of G-invariant functions on X.

On essential extensions of rings

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The aim of this talk is to give necessary and sufficient conditions for existence of universal essential left ideal extensions of rings. This is a generalization of known result for universal essential two-sided ideal extensions. Such extensions were considered by K.I. Beidar [2], J.C. Flanigan [3] and R.R. Andruszkiewicz [1].

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A hierarchy of propositional logics in Abstract Algebraic Logic

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Since the beginning of the XXth century various nonclassical propositional logics have been introduced and studied, some of which have an algebraic character. The talk will focus on the hierarchy of logics in the area of so called *Abstract Algebraic Logic* that has originated with the papers of A. Tarski, J. Łoś, R. Suszko, S. L. Bloom, R. Wójcicki, J. Czelakowski and others and which got a new impact with the introduction of the concept of a *protoalgebraic logic* by W. Blok and D. Pigozzi. Each level in the hierarchy can be characterized both semantically and syntactically via a Malcev type condition.

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$\mathcal{C}^k$-PARAMETRYZACJE ZBIORÓW DEFINIOWALNYCH W STRUKTURACH O-MINIMALNYCH

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Celem wykładu jest prezentacja następującego twierdzenia. Załóżmy, że dana jest struktura o-minimalna na ciele uporządkowanym liczb rzeczywistych $\mathbb{R}$; na przykład, struktura zbiorów semialgebraicznych. Niech E będzie zwartym definiowalnym podzbiorem $\mathbb{R}^n$ wymiaru p i niech k będzie dowolną liczbą całkowitą dodatnią. Wówczas istnieje skończona rodzina odwzorowań definiowalnych klasy $\mathcal{C}^k$

$$\varphi_\varkappa : [0,1]^p \longrightarrow \mathbb{R}^n \qquad (\varkappa \in K),$$

taka, że $E = \bigcup_{\varkappa \in K} \varphi_\varkappa([0,1]^p)$.

Banach spaces with small family of bounded operators

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I will give an overview of recent results in the theory of Banach spaces with small family of bounded operators, an area developing rapidly after the famous constructions of W.T. Gowers and B. Maurey - of a Banach space with no infinite unconditional basic sequence - and of S.A. Argyros and R. Haydon - of a Banach space with the compact-plus-scalar property (any bounded operator on the space is of the form $\lambda Id+K$ for some scalar λ and compact operator K).

The theory, focusing on Banach spaces with rather exotic structure, brings solutions to deep problems on the structure of general Banach spaces, such as Banach homogeneous space problem (if any Banach space isomorphic to any its closed infinite dimensional subspace is isomorphic to Hilbert space ℓ_2), the distortion problem (if any space isomorphic to ℓ_2 contains almost isometric copy of ℓ_2), the invariant subspace problem (if any Banach space admits a bounded operator with no non-trivial invariant subspace).

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The Łojasiewicz–Siciak condition

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The Łojasiewicz–Siciak condition is a certain estimate for the Siciak extremal function. A geometric characterization of compact semialgebraic sets in $\mathbb{C}$ satisfying the Łojasiewicz–Siciak condition will be presented.

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Bornological singularities

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Assume a gts X is a map of a region. We use the fact that it generates (among others) a bornology of small sets, which in turn may be applied to questions with 0-1 answers.

A point $x \in X$ will be called *I-singular* if it is unreachable from a point y different from x by a small I-path (i. e. x is a "forbidden city"). There are several kinds of singular points due to several unit intervals $I = [0, 1]$ understood as gtses. Moreover, a point x is a *neighbourhood singularity* if it has no small neighbourhood. Properties of sets of singular points, other kinds of points and relations between such sets are discussed, and a notion of resolution of singularities is considered.

More generally, many topological notions lead to practical applications. Examples are: nearness, distance, closure, interior, neighbourhood used to analyse digital pictures (for example in mereotopology, mathematical morphology and digital topology), L-valued bornologies used in medical diagnosis.

Some remarks on $\mathcal{GL}_n(\mathbb{F})$-invariant sets of square matrices

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Let $\mathbb{F}$ be a field and n a positive integer. In the talk, we will discuss certain subsets of the space of all $n \times n$ matrices over $\mathbb{F}$, which are invariant under the action of the full linear group $\mathcal{GL}_n(\mathbb{F})$ by conjugation (e.g., rank varieties). The focus will be on basic geometric properties and applications of rank functions.

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Hilbert space in the context of functional data analysis

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In the talk we will give some examples of functional data and then focus on a particular method of the functional data analysis (FDA) which is principal components analysis (PCA). This tool is used to analyze the covariance structure and dimension reduction. Highly dimensional data cause many difficulties while attempting to invert the covariance matrix, hence we will move to infinitely dimensional case of functional data. The starting point will be explaining the role of Hilbert spaces with reproducing kernel property (RKHS). This notion is very important because covariance operator can be represented as a kernel and might fully characterize some classes of distributions, ex. gaussian.

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The 2-adic order of generalized Fibonacci numbers

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For a given integer $k \geq 2$ we can define generalized Fibonacci numbers as

$$T_{n,k} = \sum_{i=1}^{k} T_{n-i,k}$$

for $n = k, k+1, ...$ with $T_{0,k} = 0$ and $T_{1,k} = ... = T_{k-1,k} = 1$.

We will study the 2-adic order of $T_{n,k}$. Results for $k \leq 4$ will be presented along with key ideas used in the proofs. We will also state a conjecture involving the general case. Then we will show a possible application of 2-adic valuation to solving diophantine equations of the form

$$T_{n,k} = m!,$$

where m is natural.

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Modelling financial volatility dynamics with time-inhomogeneous GARCH processes

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In finance and econometrics GARCH-type time series have been used for three decades in modelling dynamics of stock log-returns. However, fitting one specific, parametric (and usually stationary) time series model to the data with ever-growing number of entries inevitably leads to misspecification and malinference. Polymorphic structure of financial markets, where shocks of various origin impact the volatility behavior, quite quickly results in model ageing. Hence in recent years several approaches have been proposed to capture the stock returns dynamics by more flexible, time-evolving models.
Specifically, dynamics of a log-returns process $\{X_t\}_{t\in Z}$ allowing for GARCH-type conditional volatility $\sigma(t,\theta)$ is given by

$$X_t = \sqrt{g(t,\gamma)}\sigma(t,\theta)\epsilon_t$$

and driven by a noise sequence $\{\epsilon_t\}_{t\in Z}$ accounting for current news flow. We apply the above time series class – incorporating time inhomogeneity via the function $g(t,\gamma)$ – for modelling tradable volatility index (VIX) dynamics. Various types of nonstationarity can be handled by choosing appropriate g, which makes such an augmented GARCH class more flexible and applicationally useful.

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Non-parametric estimation of the conditional bivariate distribution for censored gap times

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In many survival studies, recurrent or consecutive events may occur during the follow-up study of the individuals. In this talk, we incorporate the information of a covariate into the bivariate distribution of the two gap times of interest, propose a non-parametric estimator and present its properties. We apply the new methods to Stanford Heart Transplant data.

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Efektywne metody badania punktów osobliwych odwzorowań wielomianowych

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Niech M^m będzie m–wymiarową rozmaitością. Jednym z zadań teorii osobliwości jest badanie punktów osobliwych stowarzyszonych z generycznymi gładkimi odwzorowaniami $f : M \to R^n$. Często też punktom osobliwym można przypisać pewien znak lub całkowity indeks.

W przypadku gdy odwzorowanie f jest wielomianowe a rozmaitość M jest zbiorem algebraicznym, można w pewnych przypadkach efektywnie obliczać liczbę punktów osobliwych mających określony znak lub indeks. Na wykładzie przedstawię pewne techniki takich obliczeń, wykorzystujące sygnatury form kwadratowych na odpowiednich skończenie wymiarowych algebrach, w następujących przypadkach: gdy $m = n$, gdy $n = 2m$ oraz f jest immersją, oraz gdy $m = n = 2, 3, 4$ i $M = R^m$.

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ZF-THEORY OF HAUSDORFF COMPACTIFICATIONS

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The aim of this talk is to introduce to a reasonable theory of Hausdorff compactifications in **ZF**, without the axiom of choice. A cube-compact space is a topological space which is homeomorphic with a closed subspace of a product of copies of the unit interval $[0, 1]$. A cube-compact extension of a topological space X is an ordered pair (Y, r) such that Y is a cube-compact space and r is a homeomorphic embedding of X onto a dense subspace of Y. In a model of **ZF**, a Hausdorff compactification of a Tychonoff space X can fail to be a cube-compact extension of X. An essential role in this theory is played by the following theorem: It holds true in **ZF** that every non-empty compact Hausdorff space K is the remainder of a Hausdorff compactification of an infinite discrete space.

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Bootstrap confidence bands in statistical inverse problems

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Work on construction of nonparametric confidence bands in the inverse problem setup has started only recently. Existing constructions often rely on the classical approach based on explicit limit distributions of suprema of stationary Gaussian processes, but convergence in the resulting limit theorems is rather slow. Some authors, who want to improve the finite sample coverage probabilities, use bootstrap in construction of confidence bands. The method to construct bootstrap confidence bands via the simple nonparametric bootstrap in the Wicksell's corpuscle problem will be presented. Recall that the Wicksell's stereological problem, which is a classical example of a statistical inverse problem, consists in unfolding spheres size distribution from planar sections, and arises in biology, metallurgy and many other areas. The performance of the bootstrap confidence bands will be investigated in simulation studies.

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Statistical methods for processing and signals modelling in application to technical diagnostics

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Problem of selection of informative frequency band (IFB) for local damage detection using vibration signal is often discussed in the literature. One of the approach used in this context is based on the analysis of sub-signals obtained in time-frequency representation (spectrogram) of the vibration signal. Mentioned sub-signals are analyzed using appropriate statistics (called selectors). Till now, the most popular statistic was kurtosis, one of the measure that can point out these frequency bins on time-frequency map that reveal the most impulsive nature. However for many real signals the spectral kurtosis does not give expected results because it can be sensitive for impulses not related to damage (i.e. artifacts). In our study we extend the idea of spectral kurtosis and propose not to calculate simple statistic for set of sub-signals obtained by decomposition of raw data by spectrogram but to describe each sub-signal by stochastic model that has similar properties as the analyzed time series. One of the easiest stochastic model is based on the assumption that the vector of observations contains realizations of independent identically distributed random variables. The most known distribution that can be used here is the Gaussian one. We propose a set of selectors that are constructed on the basis of the approach that distribution of sub-signals that does not correspond to the IFB is closer to Gaussian than for sub-signals related to IFB (because of the impulsive nature of those sub-signals). The mentioned selectors mostly are calculated as the distance between the empirical distribution of given sub-signals and the theoretical Gaussian distribution. We mention here selectors based on the Jarque-Bera and Kolmogorov-Smirnov statistics as well as selectors based on the quantile-quantile plot. However the approach based on the modelling of sub-signals from time-frequency map by Gaussian distribution can be extended to more general class of distributions, namely stable one. The stable distributions in the literature are considered as an extension of the classical Gaussian one. The class of this distributions are especially important in the context of modelling of data with impulsive nature but it should be mentioned that for stability parameter equal to 2 the stable distribution reduces to Gaussian one. Therefore they can be used also for sub-signals corresponding to IFB as well as not related to informative frequency band. Similar as in case of spectral kurtosis or Gaussian distribution based approach also here after application of based selector we obtain distribution of the stability parameter versus frequency, that provides similar picture as spectral kurtosis or Gaussian-based selectors. For simple signals, using our approach is not reasonable because the classical methods give similar results with smaller computational cost. However, it will be shown that for some real world examples our approach is better, especially when in the signal there are observable incidental impulses not related to damage. However application of stable distribution requires novel methods for analysis of the signal. Especially it is related to the fact that for stable distribution the classical measures of dependence as autocovariance or autocorrelation cannot be used because for most of the parameters of stable distribution they are not properly defined. Therefore we consider here alternative measures. On the basis of the time-frequency maps constructed on the basis of those measures we can enhance the classical spectrogram in order to detect impulsive nature of signals with complex nature. This talk is a synthesis of few years research activity related to development of new mathematical methods and applications of existing advanced mathematical modeling techniques for technical diagnostics of mining machines.

Singularity of fibers of singular sets

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The general theorem on fibers of singular sets will be presented. Let D_1 be a connected σ-compact Josefson manifold and let D_2 be a σ-compact complex manifold. Let $\Omega \subset D_1 \times D_2$ be a domain and let $M \subset \Omega$ be a singular set with respect to the family $\mathcal{F} \subset \mathcal{O}(\Omega \setminus M)$ such that the set $\{a_1 \in D_1 : \text{ the fiber } M_{(a_1,\cdot)} \text{ is not pluripolar}\}$ is pluripolar in D_1. We show that there exists a pluripolar set $Q \subset D_1$ such that for every $a_1 \in \pi_{D_1}(\Omega) \setminus Q$ the fiber $M_{(a_1,\cdot)}$ is singular in $\Omega_{(a_1,\cdot)}$ with respect to the family $\mathcal{F}_a := \{f(a_1,\cdot) : f \in \mathcal{F}\}$.

It generalizes a well known results of Jarnicki and Pflug from [1] and [2] to the case of complex manifolds and is one of the key properties used in the proof of strong theorems concerning extensions of functions separately holomorphic on different kinds of generalized crosses with singularities on Stein manifolds - see [3].

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Lower and upper bounds for solutions of the equation $x^m \equiv a \pmod n$

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Let n, m and a be integers such that $n \geq 2$, $m \geq 2$ and $(a, n) = 1$.
We say that a is a $m-$th power residue modulo n if there exists an integer x such that

$$x^m \equiv a \pmod n. \tag{1}$$

Let $C(n)$ denote the multiplicative group consisting of the residue classes mod n which are relatively prime to n.
Let $s(n, m, a)$ be the smallest solution of the equation (1) in the set $C(n)$.
Let $t(n, m, a)$ be the largest solution of the equation (1) in the set $C(n)$.
We will give upper bound for $s(n, m, a)$ and lower bound for $t(n, m, a)$.

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Some recurrence sequences of natural numbers modulo primes

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Let $K = (k_n)_{n=0}^{\infty}$ be an increasing sequence of natural numbers such that $k_0 = 0$. We define a new sequence $(b_K(n))_{n=0}^{\infty}$ by the following recurrence formula:

$$\begin{cases} b_K(k_n) = b_K(k_n + 1) = \ldots = b_K(k_{n+1} - 1), \\ b_K(k_n) - b_K(k_{n-1}) = b_K(n). \end{cases}$$

We do not specify the first values of this sequence. They can be equal to any non–negative integers.

We also introduce the lower degree of the sequence K:

$$\underline{\deg} K := \inf\{\ d \in \mathbb{N} \mid k_n - k_{n-1} = d \text{ for infinitely many } n \in \mathbb{N}\ \}.$$

The aim of this talk is to prove that if p is a prime number such that $3 \leq p \leq \underline{\deg} K$, then either p divides $b_K(n)$ for all but finitely many numbers $n \in \mathbb{N}$ or any residue class modulo p appears in the sequence $(b_K(n) \mod p)_{n=0}^{\infty}$ infinitely many times. We also discuss some possible generalisations of this fact.

As applications of the above theorem we get the results concerning sequences of numbers of m–ary partitions and m–ary partitions with no gaps. Congruence properties of these sequences were intensively studied by several authors, for example by Churchhouse (1969), Rödseth (1970), Andrews (1971), Gupta (1971, 1972), Dirdal (1975), Rödseth and Sellers (2001), Alkauskas (2003), Andrews, Fraenkel and Sellers (2015, 2016).