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Workshop on Modern Applied Mathematics PK 2017

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prof. dr hab. Aleksander Iwanow (Silesian University of Technology)
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prof. dr hab. Ryszard Szwarc (University of Wrocław)

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Introduction

Workshop on Modern Applied Mathematics PK 2017 is the sixth edition of an annual conference on modern mathematics organized by the Institute of Mathematics of the Faculty of Physics, Mathematics and Computer Science, Tadeusz Kościuszko Cracow University of Technology.

The Conference aims to present new results, to promote and to bring together researchers in the different research areas of mathematics and influence more cooperation among scientists working in mathematics. This conference will provide a unique forum for exchanging ideas and in-depth discussions on different aspects and different branches of mathematics.

You can find detailed information about conference on the site:

`www.wmam.pk.edu.pl`

I would like to thank professors Dariusz Dereniowski, Zbigniew Hajto, Aleksander Iwanow, Anna Pajor, Andrzej Schinzel, Daniel Simson and Ryszard Szwarc for accepting invitation to give a lecture during the conference and all participants for interest in the fourth edition of our conference and scientific research in the field of mathematics.

The conference is under the media patronage of Welcome Cracow.

I would like to express my thanks to Board of Directors and the Administration of the Institute of Mathematics as well as the staff of the Institute for their friendliness and support for conference organization.

On the behalf of the organizing committee
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Plenary Lectures

Searching graph structures

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In this talk we discuss some searching problems when generalized to graph structures. The point of departure for these problems is the classical binary search in a sorted array. A more general scenario then to consider is a partial order instead of a sorted array as a structure to be searched [1]. We discuss the problems from an algorithmic perspective and also point out several equivalent formulations, including edge ranking [2] and tree-depth [3].

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Jacobian Problem and strongly normal extensions of differential fields

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In 2003 Jerald Joseph Kovacic published his very important work "The differential Galois theory of strongly normal extensions" [1] in which he reconstructed Kolchin's fundamental ideas of differential Galois theory in the modern setting of differential algebra. In the talk, following the philosophy of Kovacic I will present a differential version of the theory of polynomial automorphisms starting from Picard-Vessiot theory [1] and will discuss deep relationship with tensor products and strongly normal extensions. Finally I will mention some recent computational results on polynomial automorphisms and Jacobian Conjecture obtained in collaboration with Elżbieta Adamus, Paweł Bogdan and Teresa Crespo [3].

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Quantum circuits, decidability and continuous logic

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Continuous logic is the basic model theoretic tool for Hilbert spaces and $\mathbb{C}^*$-algebras. This suggests that quantum circuits, quantum automata and quantum computations in general can be defined in appropriate continuous structures and studied by means of continuous logic. In my talk I will present an attempt of this approach. The main results concern decidability of continuous theories of classes of dynamical qubit spaces. They are motivated by algorithmic problems in quantum automata.

Bayesian analysis of short- and long-run relationships among prices on financial markets

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Widely-used models for financial time series are based on conditional heteroscedasticity processes. The most popular ones are the Generalized Autoregressive Conditional Heteroskedastic (GARCH) and the Stochastic

Volatility (SV) processes, though the latter are rarely used. However, some research has shown that apart from the fact that the variability of financial time series (measured, for example, by the conditional covariance matrix) varies over time, on financial markets there exist long-term relationships. Therefore, it appears essential to construct such models in which the possible presence of long-run relationships and time-variable volatility are simultaneously taken into account.

Recently, new multivariate time series models which integrate the VEC representation of a VAR structure with stochastic volatility have been proposed by Pajor and Wróblewska (2017) and by Osiewalski and Osiewalski (2013, 2016). Bayesian Vector Error Correction - Stochastic Volatility (VEC-SV) models enable us to capture the long-run relationships among processes, and also to formally examine the presence of time-variation in the conditional covariance matrix. In the VEC-SV models proposed by Pajor and Wróblewska (2017) stochastic volatility is represented by either the Multiplicative Stochastic Factor (MSF) process or by the MSF-SBEKK specification. In the VEC-GMSF-SBEKK models considered by Osiewalski and Osiewalski (2016) stochastic volatility is represented by generalizations of the MSF-SBEKK structure.

The purpose of the talk is to discuss the properties of the Bayesian VEC-MSF models in context of modelling financial time series. Another purpose of this talk is to compare the forecasting performance of three types of model specifications: the VEC-MSF model with constant conditional correlations, the VEC-MSF-SBEKK model with varying conditional correlations, and the VEC model with constant conditional covariance matrix.

Based on daily quotations on three major exchange rates:
PLN/EUR, PLN/USD and EUR/USD, the predictive capacity of the models under consideration is compared. By modelling the exchange rates (where, on the ground of lack of arbitrage opportunities, the cointegrating vector may be assumed to be known a priori) it will be possible to show how important for prediction it is to take into account a long-run relationship in financial data. The main criterion used in this study for drawing this comparison is the predictive Bayes factor. Out-of-sample analysis with the use of cumulative predictive Bayes factors shows that introducing long-run relationship into the model improves its predictive performance.

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On factors of Stern polynomials

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Klavžar, Milutinović and Petr [2] defined in 2007 the sequence of polynomials $B_n(x)$ by the formulae:

$$B_0(x) = 0, \quad B_1(x) = 1,$$
$$B_{2n}(x) = xB_n(x), \quad B_{2n+1}(x) = B_n(x) + B_{n+1}(x).$$

M. Ulas [3] conjectured in 2011 that $x + 1$, $2x + 1$, $3x + 1$ are the only primitive linear polynomials with integral coefficients and the coefficients positive that divide infinitely many $B_n(x)$; the conjecture was proved in 2014 by M. Gawron [1]. He proved besides that the lower natural density of n such that $2x + 1|B_n(x)$ or $3x + 1|B_n(x)$ is 0. The question of density will be considered in the lecture for arbitrary irreducible polynomials.

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Representations of Algebras, Mesh Quivers, Singularities of Tame Type and Birkhoff Problems for Nilpotent Linear Operators

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One of the aims of this talk is to explain an interrelation between the classification of singularities of algebraic surfaces, the local algebras of singular points, a use of the category coh-$\mathbb{X}$ of coherent sheaves on an algebraic variety X and its derived category $\mathcal{D}^b$(coh-$\mathbb{X}$) in the sense of Grothendieck-Verdier (1960), a use of module categories mod R of finite-dimensional algebras R and their derived categories $\mathcal{D}^b$(mod R) . In particular, we discuss tame representation type hypersrurface singularities of the type $f = x_1^a + x_2^b + x^m$, $a, b, m \geq 2$, in relation with m-nilpotent linear mono-representations of the two-flag poset I_{ab} and the category coh-$\mathbb{X}_{(a,b,m)}$ of coherent sheaves over the weighted projective line $\mathbb{X}_{(a,b,m)}$ in the sense of Geigle-Lenzing, see [6]. Illustrative examples will also be presented.

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The Order of indeterminate moment problems

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For a probability measure μ supported in a bounded interval the moments

$$m_n = \int_{-\infty}^{\infty} x^n d\mu(x)$$

determine this measure. In case the support of μ is unbounded the situation is much more complex. There are measures for which the moments determine them, for example

$$d\mu(x) = \pi^{-1/2} e^{-x^2}\, dx.$$

On the other hand some sequences of moments m_n correspond to many measures on the real line. The problem of determinacy can be described in terms of polynomials orthonormal with respect to the inner product

$$(f,g) = \int_{-\infty}^{\infty} f(x)\overline{g(x)} d\mu(x).$$

The theory goes back to Marcel Riesz and Rolf Nevanlinna.

On the other hand this problem is associated with essential self-adjointness of certain operators in Hilbert space, so called Jacobi matrices.

We are going to present elements of the theory and some recent results (joint with Christian Berg from Copenhagen) connecting properties of Jacobi matrices with summability properties of orthonormal polynomials.

References

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Contributed Talks

The Hodge numbers four Kummer type Calabi-Yau manifolds in arbitrary dimension

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In [1] S. Cynk and K. Hulek constructed Kummer type Calabi-Yau manifolds in arbitrary dimension as a quotient of product of elliptic curves by a group action. These varieties are supposed to satisfy one of the C. Voisin's conjectures. We will compute Hodge numbers of these varieties using Chen-Ruan cohomology ([2]).

References

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Period integrals of rigid double octic Calabi-Yau threefolds

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In his PhD thesis [2] C. Meyer gave equations of 11 rigid double octic Calabi–Yau 3–folds and the listed associated modular forms. I will present numerical computation (MAPLE) of some period intergrals and discuss relation between the lattice they generate and special L–values of the modular forms.

Joint work with Duco van Straten (Jahonnes Gutenberg–Universität, Moguncja, Niemcy)

References

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Number of solutions of equations over finite fields

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Consider a polynomial f in n variables with coefficients in a finite field with q elements of characteristic p. In the talk I will discuss an algorithm for counting the number of solutions $f = 0$ over the base field and all finite extensions of the base field. This leads to the zeta function of the hypersurface defined by f. By Dwork rationality theorem such a zeta function is always the quotient of two polynomials with integer coefficients. The algorithm ([1]) is based on the specific deformation method, which allows us to control the behaviour of the zeta function of one-parameter family of hypersurfaces. We will make use of the Lefschetz fixed point formula and properties of the zeta function known as the Weil conjectures. The motivation comes from the modularity problem of Calabi-Yau varieties ([2]).

References

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Associative words in some metabelian groups of exponent 2p

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The talk will be about associative words and group words in groups. Let F_2 be a free group, freely generated by x, y. A word $w(x, y) \in F_2$ is associative in a group G if the operation $a * b = w(a, b)$, $a, b \in G$ is associative and $w(x, y)$ is called a group word in G if the algebra $(G, *)$ is a group. It is known that in free groups and in abelian groups the only associative words are $1, x, y, xy, yx$. Płonka in 2013 described associative words and group words in the symmetric group S_3 [1].

I shall talk about the recent work joint with Witold Tomaszewski, in which we describe associative words and group words in groups which are extensions of an abelian group of odd prime exponent by a group of exponent 2.

References

[1] Płonka E., Associative words in the symmetric group of degree three, *Algebra and Discrete Mathematics* **15**(1), (2013), 83-95.

[2] Duda K., Tomaszewski W., Associative words in the varieties $\mathfrak{A}_p\mathfrak{A}_2$, in preparation.

Modified Zhu condition for 1-edge hamiltonian graphs

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We consider only finite graphs without loops and multiple edges. For a 3-connected graph G on n vertices and minimum degree $\delta < \frac{n}{2}$, we prove that if for all vertices x, y such that $\mathrm{d}(x) = \delta$ and $\mathrm{d}(y) < \frac{n+1}{2}$ we have $xy \in \mathrm{E}(G)$, then G is 1-edge hamiltonian.

References

[1] E. Flandrin and A. Marczyk and H. Li and M. Woźniak, A note on a new condition implying pancyclism, *Discussiones Mathematicae* **21** (2001) 137 — 143.

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The hidden beauty and applications of Böröczky's configurations

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Böröczky's configuration is a system of lines constructed due to the certain combinatorial problem. In the talk we will discuss some applications and properties of these configurations and their esthetic advantages.

Portfolio dynamic via copula with composite likelihood method

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We propose a method of modelling financial portfolio dynamic using copulas and composite likelihood method. Copulas allow to measure interdependence between financial instruments and to build an efficient investment portfolio. CLM makes the fit computationally faster and comparably efficient as compare to maximum likelihood method.

References

[1] Engle, Robert, Dynamic Conditional Correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models, *Jurnal of Business & Economic Statistics*, vol. 20, no 3, p.339–350, Taylor & Francis, 2002.

[2] Fermanian, Jean–David; Malongo, Hassan, On the stationarity of dynamic conditional correlation models, *Econometric Theory*, vol. 33, no 3, p. 636–663, Cambridge University Press, 2017.

[3] Varin, Cristiano; Reid, Nancy; Firth, David, An overview of composite likelihood methods, *Statistica Sinica*, p. 5-42, JSTOR, 2011.

Nonlinear singular integral operators depending on two parameters

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The aim of the talk is to present approximation properties of the nonlinear singular integral operators depending on two parameters.

We give theorems on the degree of approximation of functions by these operators. We also present specific examples of nonlinear singular integral operators for which the main theorems apply.

References

[1] H. Karsli, Convergence and rate of convergence by nonlinear singular integral operators depending on two parametres, *Appl. An.* **55**: 781–791 (2006).

[2] H. Karsli and V. Gupta, Rate of convergence of nonlinear integral operators for functions of bounded variation, *Calcolo* **45**: 87–98 (2008).

[3] T. Świderski and E. Wachnicki, Nonlinear singular integrals depending on two parameters, *Comment. Math.* **40**: 181–189 (2000).

Tensor representations and regularity of linear maps of algebras

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One of three main aspects of theory of differentiation is local linearization of mappings between topological modules. This prompts to take a closer look at linear mappings themselves. When we are given additional structure of an algebra the question arises about a possibility of representation of linear maps with use of the multiplication operation. Such a representation can be achieved by generalizing the method known from quaternionic analysis, giving us a representation of a ring $A \otimes \overleftarrow{A}$ in an abelian group $\mathcal{L}(D; B, A)$, where $\overleftarrow{A}$ denotes the opposite ring of A. However, unlike in quaternionic case, this representation need not always be faithful. This forces us to take a quotient of the algebra $A \otimes \overleftarrow{A}$. I have put forward a conjecture about the form of this quotient algebra, half of which unfortunately turned out to be false, the other half still being unresolved. During the talk the examples will be given of algebras for which conjecture holds. I will also show how this representation allows us to distinguish a class of linear endomorphisms of algebra A - called regular maps - well behaved with respect to the multiplication operation.

Estimating the mean in the Fraction of Time Approach

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n the presentation new results on estimating the mean from a single realization of the signal will be presented. The adopted approach is quite general since it does not require ergodicity assumptions. Instead, using the concept of the relative measure, the usual characteristics of the signal are built. In the single realization approach, the problem of the estimating the mean is addressed. The Central Limit Theorem is obtained with the speed of convergence that is faster than usual for some classes of functions.

References

[1] Dehay, D. Leśkow, J. and Napolitano, A. *Functional Central Limit Theorem*, IEEE Transactions on Signal Processing, 2013

Mechanisms resulting in equilibrium in the private ownership economy

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The equilibrium in the economy is often understood as the state in which economic agents do not have motivation to change their activities on markets. The concept that during the economic evolution, producers and consumers adjusted their plans of action to equilibrium was introduced to economics by Joseph Schumpeter (Schumpeter 1912). Arrow and Debreu (Arrow, Debreu, 1954) formulated the sufficient conditions for the existence of equilibrium in the competitive economy with production and the private ownership (see Debreu, 1959; Mas-Colell et al., 1995). Their results became the basis for the general equilibrium theory. In this context, let us consider a private ownership economy in which, if economic agents managed to realize their aims at given prices, then the market clearing condition would not be satisfied. It means that there is no equilibrium in the economy under study. Such aĘsituation can occur during the last stage of the economic evolution, when innovations are absorbed into the consumersŐ plans of action. The aim of the paper is to determine, under natural economic assumptions, some mechanisms in the HurwiczŐs sense (Hurwicz, Reiter, 2006; Lipieta, Malawski, 2016) which can appear within the Schumpeterian evolution. The presented mechanisms result in equilibrium in the private ownership economy and some of them are the consequences of the creative destruction principle.

References

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[2] Debreu G. (1959) Theory of value, New York. Viley.

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nomic development, Cambridge, MA, Harvard University Press 1934 and *A Galaxy Book, New York, Oxford University Press* 1961.

Properties of formal inverses of certain 2-automatic sequences

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Let p be a prime number. Let $(a_n)_{n\in\mathbb{N}}$ be an infinite sequence with values in $\mathbb{F}_p$. Consider a formal power series $F = \sum_{n=0}^{\infty} a_n X^n$. If $a_0 = 0$ and $a_1 \neq 0$, then there exists a unique formal power series $G \in \mathbb{F}_p[\![X]\!]$, such that $F(G(X)) = X$. The sequence $(b_n)_{n\in\mathbb{N}}$ of coefficients of G is then called *a formal inverse* of the sequence $(a_n)_{n\in\mathbb{N}}$. We consider the following problem: for a given p-automatic sequence $(a_n)_{n\in\mathbb{N}}$, what can be said about the properties of its formal inverse?
This problem was studied by M. Gawron and M. Ulas in the case when the sequence $(a_n)_{n\in\mathbb{N}}$ is the Thue–Morse sequence, which is 2-automatic. In the talk, I will present the properties of the formal inverses of some other automatic sequences, mainly the Baum–Sweet sequence.

References

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[2] Maciej Gawron and Maciej Ulas, On formal inverse of the Prouhet-Thue-Morse sequence *Discrete Math.*, 339(5):1459–1470, 2016.

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p-adic Denseness of Subsets Creating a Partition of $\mathbb{N}_+$ and their Quotient Sets

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Let A be a subset of $\mathbb{N}_+$. Then the set

$$R(A) = \left\{ \frac{a}{b} : a, b \in A \right\}$$

will be called the quotient set of A. Subject of denseness of sets of the form $R(A)$, where $A \subset \mathbb{N}_+$, in the set of positive real numbers was widely studied. Despite this, the issue of denseness of quotient sets in p-adic fields has many open questions. The first comprehensive study of this problem can be found in the paper [1]. Its authors stated a question about existence of sets A and B such that $A \cup B = \mathbb{N}_+$ and $R(A)$, $R(B)$ are not dense in $\mathbb{Q}_p$ for any prime number p.

The aim of the talk will be presenting p-adic topology on $\mathbb{Q}$, arithmetic interpretation of being a dense set in $\mathbb{Q}_p$ and giving the results which allow us to give the negative answer to the mentioned question.

This is a joint work with Carlo Sanna.

References

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Holomorphic polyhedra and polynomial approximation

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We will discuss an estimate for the Siciak extremal function. As an application we will present some results describing how fast holomorphic functions defined in a neighbourhood of a compact holomorphic polyhedron can be approximated uniformly on this polyhedron by complex polynomials.

References

[1] R. Pierzchała, Semialgebraic sets and the Łojasiewicz–Siciak condition, *J. Anal. Math.* 129 (2016) 285–307.

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Linear systems in symmetrized tropical algebra

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The tropical algebra $\mathbb{R}_{max} = (\mathbb{R} \cup \{-\infty\}, \max, +, -\infty, 0)$ is a semifield with operations $\oplus = \max$, $\otimes = +$. The symmetrized tropical algebra $\mathbb{S}$ is a quotient dioid obtained from some algebra structure on $\mathbb{R}^2_{max}$, where $\ominus$ is defined. Elements can have three possible signs: $\oplus$ (plus), $\ominus$ (minus) and $^\bullet$ (balanced), hence $\mathbb{S} = \mathbb{S}^\oplus \cup \mathbb{S}^\ominus \cup \mathbb{S}^\bullet$ as sets. The neutral elements for binary operations $\oplus, \otimes$ are denoted by ε, e. We get a natural balance relation: $a \nabla b$ iff $a \ominus b \nabla \varepsilon$ iff $a \ominus b \in \mathbb{S}^\bullet$. Natural operations on matrices have their tropical counterparts. Linear systems of equations over $\mathbb{R}_{max}$ of the form $A \otimes X \oplus B = C \otimes X \oplus D$ have their counterparts as linear systems of balances over $\mathbb{S}$ of the form $(A \ominus C) \otimes X \nabla (D \ominus B)$. We discuss how using the tropical version of the Moore-Penrose (pseudo)inverse of a matrix could help with finding solutions of such systems.

References

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Some applications of the Bayes theorem

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In this talk we will discuss some of the modern uses of the Bayes theorem. We will focus on presenting its usefulness in the analysis of profitability of production, analysis of mammographic findings and construction of anti-spam filter. In particular, we will show an individually created example of how an anti-filter using the Bayes theorem can be applied.

References

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Vibration analysis by functional methods

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Vibration signals sampled with a very high frequency (20 kH) constitute a basic source of information about machine behavior. Nevertheless, a big dimensionality of data set and its strong contaminations cause difficulties with detection of frequencies specific for a particular local damage. Spectral analysis tools for example spectrograms are not sufficient to distinct IFB - informative frequency band. Hence, as a counterpart to discrete methods, we propose functional approach to solve that problem. New data set becomes a collection of continuous random functions. We apply techniques of the functional data analysis (FDA), for example functional principal components analysis to reduce dimensionality. Next, we propose a new measure for IFB based on scores of FPC and try to choose the most optimal reduced model.

References

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Liczby quasi-Fibonacciego

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Aim of the speech is to present the original, multi-parametric generalizations of the Fibonacci numbers and the Lucas numbers. In fact, these generalizations are not the numbers any more, they are the real polynomials of several variables. These polynomials have been obtained on the way of searching for the most general algebraic methods enabling to generate the identities for the Fibonacci and the Lucas numbers. Such approach resulted in defining infinitely many different numerical systems named the quasi-Fibonacci numbers of the respective order. We emphasize that the quasi-Fibonacci numbers generalize many other known numerical systems (we confront them with the sequences included in OEIS).

Until now, only the one-parametric quasi-Fibonacci numbers of order n were discussed. It appears that it is proper to consider the numbers depending on $0.5\varphi(n) - 1$ parameters for natural odd n. Obviously $\varphi(\cdot)$ is here the Euler function. Case $n = 5$ is represented by the so-called δ-Fibonacci numbers - only the one-parameter numbers, well-known in literature. In my speech I will focus my attention on two (the only possible) two-parametric δ-Fibonacci numbers. I will present the basic properties of these numbers, including the summation and reduction formulae. The attractive possibilities for applications of these numbers will be also discussed. Let us notice that such applications of one-parametric numbers, that is of the mentioned δ-Fibonacci numbers, enable, among others, to reduce the calculations of type $F_{k^r} \to F_k$ for r steps (which resulted, among others, in solving some W. Webb problem).

On the least significant nonzero digits of $n!$ in base b

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In this talk we will present some results concerning the sequence $\{\ell_{b,k}(n!)\}_{n\geq 0}$ of strings of k least significant nonzero digits of $n!$ in a given base $b \geq 2$. In particular, we will show for which bases b the sequence $\{\ell_{b,k}(n!)\}_{n\geq 0}$ is automatic and find a uniform morphism generating it. This description will allow to compute how often a given string of k digits from the set $\{0, 1, ..., b-1\}$ appears in $\{\ell_{b,k}(n!)\}_{n\geq 0}$. Specific cases have already been studied, for example $b = 2, k = 3$ by Deshouillers and Luca [1] and $b = 12, k = 1$ by Deshouillers and Ruzsa [2].

References

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Skewness and Asymmetry in Generalized Gaussian Distribution Class

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The talk will focus on extensions of Generalized Gaussian Distribution – GGD (also known as Exponential Power Distribution – EPD, or Generalized Error Distribution – GED) admitting skewness and asymmetry, observed empirically eg. in econometric time series. The novel parametrization proposed by Zhu and Zinde-Walsh (2009) is more flexible as it allows for asymmetric left– and right–tail behavior. Hence its versatile applicational potential in modelling financial data. Estimation techniques will also be addressed together with risk management quantitative aspects. Moreover, further most up-to-date extension allowing for multimodality will be mentioned.

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On the containment problem

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Let I be a homogeneous ideal in the ring of polynomials over a field. The containment problem is concerned with the characterization of the set of pairs (m, r) such that there is the containment

$$I^{(m)} \subset I^r, \tag{1}$$

where $I^{(m)}$ is the m-th symbolic power of I and I^r is the ordinary r-th power of I.

Celebrated results of Ein, Lazarsfeld and Smith in characteristic 0 (asymptotic multiplier ideals) and Hochster and Huneke in positive characteristic (tight closures) give a uniform answer: the containment in (1) holds for all $m \geq r \cdot e$, where e is the big height of I. In particular in the ring of polynomials in $(N+1)$ variables, the containment in (1) holds for all

$$m \geq N \cdot r \tag{2}$$

and all homogeneous ideals I. In my talk I will report on various conjectural attempts to weaken the condition in (2) and discuss examples exhibiting limits to such attempts. The talk is based on a joint paper with Justyna Szpond [1].

References

[1] T. Szemberg and J. Szpond, On the containment problem. *Rend. Circ. Mat. Palermo* **66**: 233–245 (2017).

Asymptotyczne wartoćci krytyczne i lokalna trywializacja Nasha

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Niech $X \subset \mathbb{R}^n$ będzie domkniętą podrozmaitością Nasha. Niech $f : X \to \mathbb{R}^k$ będzie odwzorowaniem Nasha. Pokażemy, że poza zbiorem swoich uogólnionych wartości krytycznych K_f, odwzorowanie f jest lokalnie trywialne w sensie Nasha, tzn. dla każdego $y \in \mathbb{R}^n \backslash K_f$ istnieje otoczenie U punktu y, oraz istnieje dyfeomorfizm nashowski $r : f^{-1}(U) \to f^1(y) \times U$ taki, że $f_{|f^{-1}(U)} = \pi \circ r$, gdzie $\pi : f^1(y) \times U \to U$ jest rzutowaniem kanonicznym.

Quasi-cardinals in Krause's quasi-set theory for quantum mechanics

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Krause's remarkable axiomatic quasi-set theory (in abbreviation: QST) brings a more precise language for mathematics and its applications to quantum mechanics which deals with indistinguishable objects. The main aim of this talk is to give a brief introduction to QST, to show recent results on quasi-cardinals in QST and to ask open questions relevant to them. It can be proved that, under the assumption that $2^\omega = 2^{\omega_1}$, it may happen that a quasi-set can have at least two distinct quasi-cardinals, all infinite and one of them countable. A conclusion from this observation is that Krause's primitive notion of a quasi-cardinal introduced in [2] is inaccurate in QST and there does not exist an accurate definition of a countable quasi-set. It is necessary to modify QST and to search for adequate concepts of quantities of objects inside quasi-sets, taking into consideration results of [1].

References

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On some cancellation algorithms

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Definiuję $b_f(n)$ jako najmniejszą $d \in \mathbb{N}$ taką, że liczby $f(n_1, n_2, \ldots, n_m)$, gdzie $n_1 + n_2 + \ldots + n_m \leq n$ są niepodzielne przez d. Dla wybranych funkcji $f : \mathbb{N}^m \longrightarrow \mathbb{N}$ znajdę wartości elementów ciągu $(b_f(n))_{n\in\mathbb{N}}$ lub podam inną jego charakteryzację.
Browkin i Cao [1] pokazali, że dla funkcji $f : \mathbb{N}^2 \ni (n_1, n_2) \longrightarrow n_1^2 + n_2^2 \in \mathbb{N}$, ciąg $(b_f(n))_{n\in\mathbb{N}}$ to rosnący ciąg kolenych bezkwadratowych liczb naturalnych, będących iloczynami liczb pierwszych przystających do 3 modulo 4. W swoim refaracie przedstawię wyniki z trzech prac [2],[3],[4],[5] dotyczące następujących funkcji:

$$f_1(n_1) = n_1^k, k \geq 2,\ f_2(n_1, n_2, \ldots, n_m) = n_1 n_2 \cdot \ldots \cdot n_m,\ m \geq 2,$$

$$f_3(n_1, n_2, n_3) = n_1^2 + n_2^2 + n_3^2,\ f_4(n_1, n_2, n_3, n_4) = n_1^2 + n_2^2 + n_3^2 + n_4^2,$$

$$f_5(n_1, n_2) = n_1^j + n_2^j,\ j > 3, \text{ nieparzyste},$$

$$f_6(n_1) = n_1!, f_7(n_1) = n_1!!, f_8(n_1) = n_1 b^{n_1}, \text{dla ustalonego } b \in \mathbb{N},$$

$$f_9(n_1, n_2) = 2^{n_1+n_2} - 2^{n_1},$$

$f_{10}(n_1) = F_{n_1}$, gdzie F_n oznacza n-tą liczbę Fibonnaciego,

$$f_{11}(n_1, n_2) = od(n_1 + n_2) - od(n_1),$$

gdzie $od(n)$ oznacza n-tą wstrętną liczbę „odious number".
Dla funkcji $f : \mathbb{N}^2 \ni (n_1, n_2) \to n_1^3 + n_2^3 \in \mathbb{N}$, charakteryzacja ciągu $(b_f(n))_{n\in\mathbb{N}}$ może być podana używając wielomianów permutacyjnych skończonego, przemiennego, pierścienia ilorazowego $\mathbb{Z}/m\mathbb{Z}$.

W szczególnych przypadkach funkcji f podam dolne i górne ograniczenia na wartości elementów ciągu $(b_f(n))_{n\in\mathbb{N}}$.

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On p-adic valuations of colored p-ary partitions

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Let $m \in \mathbb{N}_{\geq 2}$ and for given $k \in \mathbb{N}_+$ consider the sequence $(A_{m,k}(n))_{n\in\mathbb{N}}$ defined by the power series expansion

$$\prod_{n=0}^{\infty} \frac{1}{(1-x^{m^n})^k} = \sum_{n=0}^{\infty} A_{m,k}(n)x^n.$$

The number $A_{m,k}(n)$ counts the number of representations of n as sums of powers of m, where each summand has one among k colors.

In the talk we will present some results concerning the p-adic valuations of the numbers $A_{p,(p-1)(up^s-1)}(n)$, where $p \in \mathbb{P}_{\geq 3}$, $u \in \{2,\dots,p-1\}$ and $s \in \mathbb{N}_+$. These facts generalize the findings obtained for $p = 2$ by Gawron, Miska and Ulas in [1].

The talk is based on joint work with Maciej Ulas.

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Appendix

Goodness-of fit for randomly censored data based on maximum correlation

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The maximum correlation, ρ^+, is the Frechet-Hoeffding upper bound of the correlation coefficient corresponding to the bivariate distribution function $H^+(x, y) = \min(F_1(x), F_2(y))$. It is a measure of agreement between the distribution functions, F_1 and F_2, since $\rho^+ = 1$ if and only if $F_1 = F_2$ up to a scale and location change. We study the goodness-of-fit test based on ρ^+ in the context of randomly censored data under a general censoring mechanism and also in the case of the Koziol-Green model. We prove the asymptotic properties of the proposed test statistic. Additionally, we present an extensive simulation study on the empirical power which shows a good performance of our approach and its advantages over the most famous Pearson-Type test.

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