KRAKÓW 16 – 18 listopada 2018

WORKSHOP ON MODERN APPLIED MATHEMATICS PK 2018

Book of Abstracts

Kraków

Workshop on Modern Applied Mathematics
PK 2018

November 16-18, 2018

Kraków, Poland

Scientific Committee

dr hab. Ludwik Byszewski, prof. PK — Chairman
prof. dr hab. Orest D. Artemovych
prof. dr hab. Sławomir Cynk
prof. dr hab. Piotr Grzeszczuk
prof. dr hab. Jan Krempa
prof. dr hab. Kamil Rusek
prof. dr hab. Tomasz Szemberg
dr hab. Dariusz Cichoń
doc. dr hab. Piotr Jakóbczak
dr hab. Włodzimierz Jelonek
dr hab. inż. Jacek Leśkow, prof. PK
dr hab. Ihor Mykytyuk
dr Grzegorz Gancarzewicz
dr Marcin Skrzyński

Organizing Committee

dr Grzegorz Gancarzewicz — Chairman
dr Magdalena Grzech
dr Monika Herzog
dr Beata Kocel-Cynk
dr Kamil Kular
dr Marcin Skrzyński

Invited Speakers

dr Magda Lemańska (Gdańsk University of Technology)
prof. Tadeusz Mostowski (University of Warsaw)
prof. Krzysztof Nowak (Jagiellonian University)
prof. RNDr. Zdeněk Ryjáček, DrSc. (University of West Bohemia)

Contents

Introduction

Workshop on Modern Applied Mathematics PK 2018 is the seventh edition of an annual conference on modern mathematics organized by the Institute of Mathematics of the Faculty of Physics, Mathematics and Computer Science, Tadeusz Kościuszko Cracow University of Technology.

The Conference aims to present new results, to promote and to bring together researchers in the different research areas of mathematics and influence more co-operation among scientists working in mathematics. This conference will provide a unique forum for exchanging ideas and in-depth discussions on different aspects and different branches of mathematics.

You can find detailed information about conference on the site:

`www.wmam.pk.edu.pl`

I would like to thank professors Magda Lemańska, Tadeusz Mostowski, Krzysztof Nowak and Zdeněk Ryjáček for accepting invitation to give a lecture during the conference and all participants for interest in the fourth edition of our conference and scientific research in the field of mathematics.

The conference is under the media patronage of Welcome Cracow.

I would like to express my thanks to Board of Directors and the Administration of the Institute of Mathematics as well as the staff of the Institute for their friendliness and support for conference organization.

On the behalf of the organizing committee
Grzegorz Gancarzewicz

List of Participants

Orest D. Artemovych, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Justyna Burdyl, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Dominik Burek, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Ludwik Byszewski, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Tymoteusz Chmiel, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Mateusz Chojnacki, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Marta Ciarkowska, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Dariusz Cichoń, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Sławomir Cynk, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Justyna Czyszczoń, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Barbara Dworzańska, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Grzegorz Gancarzewicz, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Kamil Gąsior, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Kinga Głąbińska, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Karolina Góra, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Paulina Górny, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Magdalena Grzech, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Marek Janasz, Pedagogical University of Cracow, Institute of Mathematics, ul Podchorążych 2, 30-084 Kraków

Włodzimierz Jelonek, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Olena Karpel, ILTPE NASU, Kharkiv, Ukraine / AGH University of Science and Technology, al. Mickiewicza 30, 30-059 KrakĂłw Kraków,

Paweł Kita, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Weronika Kohla, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Michał Kozdęba, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Beata Kocel-Cynk, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Patrycja Kocoń, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Tomasz Kowalczyk, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Adrian Kowalski, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Grażyna Krech, Pedagogical University of Cracow, Institute of Mathematics, ul Podchorążych 2, 30-084 Kraków

Jan Krempa, University of Warsaw, Institute of Mathematics, ul. Banacha 2, 02-097 Warszawa

Krzysztof Książek, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Kamil Kular, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Karolina Lalik, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Magdalena Lampa - Baczyńska, Pedagogical University of Cracow, Institute of Mathematics, ul Podchorążych 2, 30-084 Kraków

Jacek Leśkow, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Magda Lemańska, Gdańsk University of Technology, Department of Probability and Biomathematics, ul. Gabriela Narutowicza 11/12, 80-233 Gdańsk

Agnieszka Lipieta, Kraków University of Economics, ul. Rakowicka 27, 31-510 Kraków

Angelika Łuczywo, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Kamila Majcher, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Anna Mazur, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Piotr Miska, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Tadeusz Mostowski, University of Warsaw, ul. Banacha 2, 02-097 Warszawa

Ihor V. Mykytyuk, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Ilona Mytyk, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Krzysztof Nowak, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Marta Nowicka, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Paulina Obara, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Piotr Pająk, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Katarzyna Pałasińska, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Magdalena Pawińska, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Natalia Pieróg, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Rafał Pierzchała, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Artur Piękosz, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Agata Prasełek, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Monika Purcha, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Kamil Rusek, Pedagogical University of Cracow, Institute of Mathematics, ul. Podchorążych 2, 30-084 Kraków

Zdeněk Ryjáček, University of West Bohemia, Department of Mathematics, Technická 2967/8, 306 14 Plzeň , Czech Republic

Marcin Skrzyński, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Bartosz Sobolewski, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Aleksandra Stawarz, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Tomasz Szemberg, Pedagogical University of Cracow, Institute of Mathematics, ul Podchorążych 2, 30-084 Kraków

Monika Ślusarczyk, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Oktawia Targosz, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Anna Valette, Jagiellonian University, Institute of Mathematics, ul. prof. Stanisława Łojasiewicza 6, 30-348 Kraków

Eliza Wajch, Siedlce University of Natural Sciences and Humanities, Institute of Mathematics and Physics, ul. 3 Maja 54, 08-110 Siedlce

Gabriela Wilkosz, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Dominika Wołoszyn, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Maciej Zakarczemny, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Paulina Zawiślak, Cracow University of Technology, Institute of Mathematics, ul. Warszawska 24, 31-155 Kraków

Plenary Lectures

Domination in graphs

Magda Lemańska

Gdańsk University of Technology, Gdańsk

e-mail: `magleman@pg.edu.pl`

A *dominating set* of a graph $G = (V, E)$ is a set $D \subseteq V$ such that every vertex of G is either in D or has a neighbour in D. A minimum cardinality of a dominating set of G is a *domination number* of G and is denoted by $\gamma(G)$. Except of being dominating, we can require some other properties of a set D, in terms of, for example, subraphs induced by D (such subraphs can be without isolates, have a perfect matching, be connected etc.) and consider various kinds of domination in graphs. In particular we consider classical domination, total domination and convex domination; give some general properties of domination, total domination and convex domination numbers, influence of some graph operations on these numbers and some applications.

References

[1] T. Haynes, S. Hedetniemi, P. Slater, Fundamentals of domination in graphs, Marcel Dekker, Inc., (1998)

[2] T. Haynes, S. Hedetniemi, P. Slater, Domination in graphs- Advanced Topics, Marcel Dekker, Inc., (1998)

[3] T. Haynes , M. Henning , L. Hopkins, Total domination subdivision numbers of graphs, Discussiones Mathematicae Graph Theory 24 (3) (2004) 457-467.

[4] M. Dettlaff, M. Lemańska M., S. Kosary, M.S. Sheikholeslami, The convex domination subdivision number of a graph, Communications in Combinatorics and Optimization 1 (1) (2016) 43–56.

Wokół hipotezy gradientu R. Thoma

Tadeusz Mostowski

University of Warsaw, Warszawa

e-mail: `tmostows@mimuw.edu.pl`

Referat będzie dotyczył wlasności trajektorii gradientu funkcji analitycznej rzeczywistej. Przedstawię motywacje i klasyczny wynik S. Łojasiewicza. Nastepnie sformułuję hipotezę gradientu, jej szczególne przypadki i- krótko-ideę jej dowodu. Sformułuję jej dalekie wzmocnienie: hipotezę nieoscylacji. Wreszcie podam parę nierozwiązanych problemów. Podkreslę związek tych zagadnień z wlasnościami zbiorów semi-analitycznych i ich uogólnień.

A closedness theorem and applications in geometry of rational points over Henselian valued fields

Krzysztof Nowak

Jagiellonian University, Kraków

e-mail: `nowak@im.uj.edu.pl`

My talk is devoted to geometry of algebraic subvarieties of K^n over arbitrary Henselian valued fields K of equicharacteristic zero, investigated in my recent papers [3,4,5,6]. At the center of my approach is the closedness theorem to the effect that the projections $K^n \times \mathbb{P}^m(K) \to K^n$ are definably closed maps. It enables, in particular, application of resolution of singularities in much the same way as over locally compact ground fields. Its proof uses i.a. the local behavior of definable functions of one variable and fiber shrinking, being a relaxed version of curve selection. As applications, I achieve several versions of curve selection (via resolution of singularities) and of the Łojasiewicz inequality (via two instances of quantifier elimination indicated below), extending continuous hereditarily rational functions (cf. [2] for the real algebraic version) as well as the theory of regulous functions, sets and sheaves, including Nullstellensatz and Cartan's theorems A and B (cf. [1] for the real algebraic versions). Two basic tools involved are quantifier elimination for Henselian valued fields (in the language of Denef–Pas) due to Pas and relative quantifier elimination for ordered abelian groups (in a many-sorted language with imaginary auxiliary sorts) due to Cluckers–Halupczok. Other, new applications of the closedness theorem are piecewise continuity of definable functions, Hölder continuity of definable functions on closed bounded subsets of K^n and a non-Archimedean, definable version of the Tietze–Urysohn extension theorem from [6]. In the paper [5], I established a non-Archimedean version of the closedness theorem over Henselian valued fields with analytic structure along with several applications.

References

[1] G. Fichou, J. Huisman, F. Mangolte, J.-P. Monnier, *Fonctions régulues*, J. Reine

Angew. Math. **718** (2016), 103–151.

[2] J. Kollár, K. Nowak, *Continuous rational functions on real and p-adic varieties*, Math. Zeitschrift **279** (2015), 85–97.

[3] K.J. Nowak, *Some results of algebraic geometry over Henselian rank one valued fields*, Sel. Math. New Ser. **23** (2017), 455–495.

[4] K.J. Nowak, *A closedness theorem and applications in geometry of rational points over Henselian valued fields*, arXiv:1706.01774 [math.AG] (2017).

[5] K.J. Nowak, *Some results of geometry over Henselian fields with analytic structure*, arXiv:1808.02481 [math.AG] (2018).

[6] K.J. Nowak, *A non-Archimedean version of the Tietze–Urysohn theorem over Henselian valued fields*, arXiv:1808.09782 [math.AG] (2018).

Graph properties, forbidden subgraphs and hereditary classes

Zdeněk Ryjáček

University of West Bohemia Pilsen, Czech Republic

e-mail: `ryjacek@kma.zcu.cz`

A graph property $\mathcal{P}$ is *hereditary* if, whenever a graph G has property $\mathcal{P}$, so does every its induced subgraph, and a class of graphs $\mathcal{C}$ is hereditary if $G \in \mathcal{C}$ implies $G' \in \mathcal{C}$ for every induced subgraph G' of G.

Hereditary properties and classes of graphs are known to have many "nice" properties, and graph problems that are difficult in general can be often substantially simplified by imposing a restriction to a suitable hereditary class. Among others, hereditary classes of graphs often have a "nice" characterization in terms of forbidden induced subgraphs (i.e., a graph family $\mathcal{F}$ such that $G \in \mathcal{C}$ if and only if G is $\mathcal{F}$-free), or in terms of a universal graph (i.e., a graph $\mathcal{G}$ such that $G \in \mathcal{C}$ if and only if G is an induced subgraph of $\mathcal{G}$).

In the talk, we illustrate these facts on several classical graph-theoretical problems, namely, on Hamilton-type problems, on graph colorings and on graph labelings, and we present some recent results in these fields.

Contributed Talks

Kummer type Calabi-Yau varieties with automorphism of order 6

Dominik Burek

Jagiellonian University, Kraków

e-mail: `dominik.burek@doctoral.uj.edu.pl`

Based on Cynk-Hulek method from [1] we construct complex Calabi-Yau varieties of arbitrary dimensions using elliptic curves with automorphism of order 6. Also we give formulas for Hodge numbers of varieties obtained from that construction. We shall generalize result of [2]] to obtain arbitrarily dimensional Calabi-Yau manifolds which are Zariski manifolds in characteristic $p \equiv 2 \pmod 3$ or $p \equiv 3 \pmod 4$.

References

[1] CH S. Cynk, K. Hulek, *Higher-dimensional modular Calabi-Yau manifolds*, Canad. Math. Bull. **50** (2007), 486–503.

[2] KatsuraSchutt T. Katsura, M. Shütt *Zariski $K3$ surfaces*, arXiv preprint

---

Period integrals of rigid Calabi-Yau threefolds with Picard-Fuchs operators

Tymoteusz Chmiel

Jagiellonian University, Kraków

e-mail: `tymoteuszchmiel@gmail.com`

Period integrals of a rigid Calabi-Yau threefold are expected to form a lattice that is commensurable with the lattice generated by special values of the L-function of the corresponding modular form.

The aim of this talk is to present conjectural method of numerical computation of period integrals of rigid Calabi-Yau threefolds. It is based on an analysis of the Picard-Fuchs operator of a family containing birational model of a given threefold in its closure.

This method was numerically verified for 29 one-parameter families of double covers of three-dimensional projective space, branched along a sum of eight planes, in singular points of which birational models of rigid double octics appear.

This talk is based on my bachelor thesis defended this year at the Jagiellonian University.

References

[1] S. Cynk, D. van Straten, *Periods of double octic Calabi-Yau manifolds.* arXiv:1709.09751

[2] C. Meyer, *Modular Calabi-Yau threefolds*. Fields Institute Monographs, 22. American Ma- thematical Society, Providence, RI, 2005.

Number of points of a double octic over a finite field

Sławomir Cynk

Jagiellonian University, Kraków

e-mail: `slawomir.cynk@uj.edu.pl`

I will describe an algorithm to compute the number of points of a double octic over a finite field. A double octic is a Calabi-Yau threefold constructed as a resolution of the projective space branched along an arrangement of eight planes. Double octic Calabi-Yau threefolds were introduced in [1], they are intensively studied in the context of the modularity theorem (see [2]). The Faltings–Serre–Livné method of proving modularity - which is standard especially in the case of rigid Calabi-Yau threefolds - is based on counting of points over finite fields. For a fixed prime p the numbers of points over the field $\mathbb{F}_{p^k}$ define the zeta function, which by Dwork's theorem is rational.
Presented algorithm was developed by Aleksander Czarnecki during his PhD studies

References

[1] S. Cynk, T. Szemberg, *Double covers and Calabi–Yau varieties*, Banach Center Publ. **44** (1998), 93–101.

[2] C. Meyer, Modular Calabi-Yau threefolds. Fiebilds Institute Monographs, 22. American Mathematical Society, Providence, RI, 2005.

Non-parametric regression - practical application

Kinga Głąbińska

Cracow University of Technology, Kraków

e-mail: *`kingagla@gmail.com`*

The presentation shows how a non-parametric regression model is created from scratch. At the beginning it shows what the kernel function is, what purpose it is used for and which ones are the most popular. The listener will also find the answer to the question about how different the model differs depending on the selected kernel function. Next, the smoothing parameter is discussed along with an explanation of how to select it and what results in a bad selection of such a parameter. The theoretical foundations are presented in a practical example, where each of the previously discussed aspects of regression is shown from the practical side. The life situations in which nonparametric regression is also used will also be presented

References

[1] Ricardo Cao, *Nonparametric curve estimation: an overview, Investigaciones Economicas*, vol. XXI(2), 1997, 209 – 252

[2] Julian J.Faraway, *Linear Models with R*, CRC press, 2009

[3] Wolfgang Härdle, *Applied Nonparametric Regression* , Berlin, 1994

[4] Adam Krzyzak, et al., *A Distribution-Free Theory of Nonparametric Regression*, Springer, 2002

A rational counterexample for the containment $I^{(3)} \subset I^2$

Marek Janasz

Pedagogical University of Cracow

e-mail: `marek.janasz@up.krakow.pl`

The interest in the containment relation between $I^{(3)}$ and I^2 for ideals of points in $\mathbb{P}^2$ is motivated by a question posted by Huneke around 2000. In this presentation we will show a rational counterexample to this containment. This example is interesting for the number of reasons. It is the first counterexample where all 127 intersection points coming from the arrangement are involved. There is an element from the set $I^{(3)} \setminus I^2$ which is the product of lines and the high degree, irreducible curve. Together with another simplicial line arrangement, which is identical to presented from combinatorics point of view, but differ in the dual configuration of lines, is an example that being a counterexample is independent from the combinatorics.

References

[1] Grünbaum B.: A catalogue of simplicial arrangements in the real projective plane, *ARS Mathematica Contemporanea* **2**: 1–25 (2009).

[2] Huneke, C.: Open problems on powers of ideals. Notes from a workshop on Integral Closure, Multiplier Ideals and Cores, AIM, (2006). `www.aimath.org/WWN/integralclosure/Huneke.pdf`

Ergodic invariant measures for Bratteli diagrams

Olena Karpel

ILTPE NASU, Kharkiv, Ukraine / AGH University of Science and Technology, Kraków, Poland

e-mail: **helen.karpel@gmail.com**

We study the simplex $\mathcal{M}_1(B)$ of probability measures on a Bratteli diagram B which are invariant with respect to the tail equivalence relation. Equivalently, $\mathcal{M}_1(B)$ is formed by probability measures invariant with respect to a homeomorphism of a Cantor set. We prove a criterion of unique ergodicity of a Bratteli diagram. In the case of a finite rank k Bratteli diagram B, we give a criterion for B to have exactly $1 \leq l \leq k$ ergodic invariant measures and describe the structures of the diagram and the subdiagrams which support these measures. Given a natural number l, we find sufficient conditions under which a Bratteli diagram of arbitrary rank has exactly l probability ergodic invariant measures. This is a joint work with S. Bezuglyi and J. Kwiatkowski. The talk contains the results of studies conducted by President's of Ukraine grant for competitive projects (F75/145-2018) of the State Fund for Fundamental Research.

A version of Cartan's Theorem A for coherent sheaves on real affine varieties

Tomasz Kowalczyk

Jagiellonian University

e-mail: **tomek.kowalczyk@student.uj.edu.pl**

It is known that Cartan's Theorem A does not hold in the real algebraic case. Nevertheless, we established a real version of Theorem A to the effect that for any coherent sheaf $\mathcal{F}$ on a non-singular real affine variety X there exists a multi-blowup $\sigma : \widetilde{X} \to X$ such that the pull-back sheaf $\sigma^*\mathcal{F}$ is generated by global sections on $\widetilde{X}$.

References

[1] T. Kowalczyk, *A version of Cartan's Theorem A for coherent sheaves on real affine varieties, Int. J. Math. 29 (08) (2018), 1850058.*

Minimal projection onto certain subspace of $L_p(X \times Y \times Z)$

Michał Kozdęba

Jagiellonian University, Kraków

e-mail: **michal.w.kozdeba@gmail.com**

Let X, Y, Z be Banach spaces. We show a formula for a minimal projection from $L_p(X \times Y \times Z)$ onto its subspace $L_p(X \times Y) + L_p(X \times Z) + L_p(Y \times Z)$ and its

generalization for space $L_p(X_1 \times X_2 \times \ldots \times X_n)$.
It is an extension of a result of Cheney and Light [1] who showed a formula for a minimal projection from $L_p(X \times Y)$ onto $L_p(X) + L_p(Y)$

References

[1] Cheney E. W., Light W. A. *Approximation Theory in Tensor Product Spaces,* Lecture Notes in Mathematics, Springer-Verlag, Berlin, 1985.

Approximation by positive linear operators associated with Hermite polynomials

Grażyna Krech

AGH University of Science and Technology, Kraków
e-mail: `krech@agh.edu.pl`

The aim of this talk is to present approximation properties of linear, positive operators associated with orthogonal expansions. We consider Poisson type integrals for Hermite and Laguerre expansions and study their approximation properties in the L^p space.

We also discuss some combinations of the operators presented here and compute their rate of convergence.

References

[1] G. Krech, Approximation Theorems for Positive Linear Operators Associated with Hermite and Laguerre Polynomials, *Advances in Summability and Approximation Theory*, Springer Nature Singapore Pte Ltd. 2018 – in print.

[2] G. Krech, On some approximation theorems for the Poisson integral for Hermite expansions, *Analysis Mathematica* **40**: 133–145 (2014).

[3] G. Toczek, E. Wachnicki, On the rate of convergence and the Voronovskaya theorem for the Poisson integrals for Hermite and Laguerre expansions, *J. Approx. Theory* **116**: 113–125 (2002).

Containment problem and combinatorics

Magdalena Lampa-Baczyńska

Pedagogigacl University of Cracow
e-mail: `lampa.baczynska@wp.pl`

In algebraic geometry and commutative algebra there has been recently a lot of interest in comparing usual (algebraic) and symbolic powers of homogeneous ideals. It was motivated by a following conjecture of Harbourne and Huneke (see details in [2]).

Conjecture 1. *Let $I \subset \mathbb{K}[\mathbb{P}^N]$ be the ideal of finite number of points. Then $I^{(m)} \subset I^r$ if $m \geq rN - (N-1)$.*

The research in this area was based on the ideals of points coming from the lines arrangements and the combinatorial features of these configurations of lines. Recently in [1] appeared first example showing that this path of reasoning is not necessary correct.

The purpose of this talk is to give short introduction to the problem of containment relations between powers of the ideals and survey some new results in this area.

References

[1] Farnik, Ł., Kabat, J., Lampa-Baczyńska, M., Tutaj-Gasińska, H.: Containment problem and combinatorics, to appear in Journal of Algebraic Combinatorics

[2] Szemberg, T., Szpond, J.: On the containment problem, Rend. Circ. Mat. Palermo (2) 66 (2017), 233–245.

Creative destruction as the mechanism leading to equilibrium in the competitive economy

Agnieszka Lipieta

Cracow University of Economics, Kraków

e-mail: `alipieta@uek.krakow.pl`

Joseph Schumpeter considered two different mechanisms governing two forms of the economy i.e. the circular flow and the economic development (Schumpeter, 1912). Namely, the tatonnement mechanism which moves the economic system to a state of Walras equilibrium (see (Walras, 1954)) as well as the creative destruction moving the economic system, through the imitation processes, to aĘnew equilibrium state. The creative destruction, by Schumpeter, consists of two opposite processes: innovative processes resulting in the introduction of new commodities, new technologies, new organizational structures etc. and processes of elimination of existing, outdated solutions. A state of equilibrium which earlier had been the aim of economic processes, became the initial point of further development of the economy. In the above elaborations the economic mechanism, understood as the set of rules and regularities explaining the social and economic life, played a significant role. The aim of this research is the analysis of the creative destruction by incorporating Hurwicz mechanisms (Hurwicz, 1987; Lipieta, Malawski, 2016) in a suitably modified Arrow-Debreu model (see (Arrow, Debreu, 1954).

This work is supported by National Science Centre in Poland, GRANT 2017/27/B/HS4/00343.

References

[1] Arrow K. J ., Debreu G., *Existence of an equilibrium for a competitive economy*, Econometrica 22; 265-290, 1954

[2] Hurwicz L.*Incentive Aspects of Decentralization* in Arrow, K. J., Intriligator, M. D. (eds), Handbook of Mathematical Economics, vol.3. Amsterdam, 1987

[3] Lipieta A., Malawski A. *Price versus Quality Competition: In Search for Schumpeterian Evolution Mechanisms*, Journal of Evolutionary Economics 26 (5), 2016

[4] Schumpeter J. A *Die Theorie der wirtschaftlichen Entwicklung* Leipzig, Duncker & Humblot, 1912

[5] Schumpeter J.A. *Capitalism, Socialism and Democracy*, 3th enlarged edn., Harper, New York, 1942

[6] Walras L. (1954) *Elements of Pure Economics*. Allen and Unwin. London, 1954

On numbers of permutations being products of pairwise disjoint cycles of length d

Piotr Miska

Jagiellonian University, Kraków

e-mail: `piotrmiska91@gmail.com`

In [1] T. Amdeberhan and V. Moll studied combinatorical identities, 2-adic valuations and asymptotics of numbers $H_2(n)$ of involutions of a set with n elements, i. e. permutations $\sigma \in S_n$ such that σ^2 is the identity function. Let us notice that each involution can be written as a product of pairwise disjoint transpositions. Then there is natural to ask about arithmetic properties of numbers $H_d(n)$ of permutations of a set with n elements which are products of pairwise disjoint cycles of length d (d is a fixed positive integer greater than 1). During the talk I will present some results on numbers $H_d(n)$, e. g. periodicity of sequences $(H_d(n) \pmod{p^r})_{n\in\mathbb{N}}$ where p is a prime number and r is a positive integer, p-adic valuations and properties of polynomials associated with exponential generating functions of sequences $(H_d(n))_{n\in\mathbb{N}}$.

This is a joint work with Maciej Ulas.

References

[1] T. Amdeberhan and V. H. Moll, *Involutions and their progenies*, J. Comb., 6 (4) (2015), 483-508.

Definable homotopy theory and GTS

Artur Piękosz

Cracow University of Technology, Kraków
e-mail: *apiekosz@pk.edu.pl*

The (weakly) semialgebraic homotopy theory of H. Delfs and M. Knebusch (developed in the 1980s) was extended in 2013 to a (weakly) definable homotopy theory over o-minimal expansions of real closed fields by A. Piękosz. One can hope for further generalizations: for any expansions of real closed fields or even for any expansions of ordered fields.

One of the main tools was the generalized topology of Delfs and Knebusch, which allows to glue together infinitely many definable sets. A generalized topological space induces a usual topology and many interesting bornologies (for example: the bornology of small sets). One may see a generalized topological space as a bornological universe with additional structure. We mention the problems of quasi-metrization and strict compactification of generalized topological spaces.

The category of generalized topological spaces and strictly continuous mappings, denoted by **GTS**, has many interesting full subcategories and gives a series of examples of topological constructs. The subcategories of small spaces (**SS**) and locally small spaces (**LSS**) have nice simple descriptions. The subcategory of partially topological spaces $\mathbf{GTS}_{pt}$ and its subcategories may lead to new branches of topological algebra.

References

[1] H. Federer, *Geometric measure theory*, Springer-Verlag, Berlin, 1996.

[2] M. Omladič and P. Šemrl, Matrix spaces with bounded number of eigenvalues, *Linear Algebra Appl.* **249**: 29–46 (1996).

A generalization of Bourgain's inequality

Rafał Pierzchała

Jagiellonian University, Kraków
e-mail: *Rafal.Pierzchala@im.uj.edu.pl*

We will recall a certain polynomial inequality for convex bodies due to J. Bourgain and present its generalization for subanalytic sets.

References

[1] R. Pierzchała, Remez-type inequality on sets with cusps, Adv. Math. 281 (2015) 508–552.

Cyclotomic properties of polynomials associated with automatic sequences

Bartosz Sobolewski

Jagiellonian University, Kraków

e-mail: `bartosz.sobolewski@doctoral.uj.edu.pl`

Consider a k-automatic sequence $\{a_n\}_{n\geq 0}$ and the polynomials

$$A_n(x) = \sum_{m=0}^{n-1} a_m x^m.$$

For a primitive r-th root of unity ω_r, with r coprime to k, we study the behavior of $A_n(\omega_r)$. We derive a recurrence relation of the form

$$\sum_{i=0}^{l} P_i(\omega_r) A_{n+is}(\omega_r) = 0,$$

where s is such that $k^s \equiv 1 \pmod{r}$, and $P_0, \dots, P_l$ are polynomials independent of n. This is a generalization of a result by Brillhart, Lomont and Morton [1] concerning the Rudin–Shapiro sequence.

We show that in general l can be bounded from above by the number of states in the automaton generating $\{a_n\}_{n\geq 0}$ and provide a sharper bound for some special cases.

We also study the integrality of the coefficients $P_0(\omega_r), \dots, P_l(\omega_r)$.

References

[1] J. Brillhart, J.S. Lomont, and P. Morton, Cyclotomic properties of the Rudin–Shapiro polynomials, *J. Reine Angew. Math.* **288**: 37–65 (1976).

Model Kermacka - McKendricka

Oktawia Targosz

Cracow University of Technology

e-mail: `oktawia.targosz96@gmail.com`

Model type SIR. Kermack- McKendrick model assumes the division of a constant number of individuals in the population into three groups: 1) Healthy and susceptible individuals (susceptible), 2) Infested individuals (infected), 3) Individuals who have acquired permanent immunity or have been removed from the population, i.e. they have no further connection with the development disease (resistant) Further activities consist in examining the model using mathematical methods and statements describing their operation

References

[1] R. Rudnicki, *Modele i metody biologii matematycznej* Instytut Matematyczny Polskiej Akademii Nauk, Warszawa 2014,s.85-89

[2] W.O. Kermack, A.G. McKendric, *A contribution to the mathematical theory of epidemics*, Proc.Roy,s.713-714

Łojasiewicz inequality at singular point

Anna Valette

Jagiellonian University, Kraków
e-mail: `anna.valette@gmail.com`

In this talk we will prove a generalized version of Łojasiewicz inequality. The famous Łojasiewicz inequality asserts that if f is a $\mathcal{C}^1$ globally subanalytic function in a neighborhood of a point $a \in \mathbb{R}^n$ there is a neighborhood U of a and a rational number $\theta \in [0, 1)$ as well as a constant C such that $|f(x) - f(a)|^\theta \leq C|\nabla_x f|$ for $x \in U$ (where $\nabla_x f$ stands for the gradient of the function f at x). We will give an inequality of the same type that applies to the case where a is not an interior point of the domain of f.

A famous theorem of Glicksberg on Čech-Stone compactifications of products fails in a model of ZF

Eliza Wajch

Siedlce University of Natural Sciences and Humanities, Siedlce
e-mail: `eliza.wajch@gmail.com`

If a Tychonoff space X has its Čech-Stone compactification, then βX stands the Čech-Stone compactification of X. One of the most famous theorems of Glicksberg that holds true in **ZFC** asserts that if X and Y are pseudocompact Tychonoff spaces such that $X \times Y$ is pseudocompact, then $\beta X \times \beta Y$ is the Čech-Stone compactification of $X \times Y$ (cf. [1]). A proof to a theorem of [3] that the above-mentioned Glicksberg's theorem is false in **ZF**-model $\mathcal{M}37$ of [2] will be shown. Other relevant new problems and applications of amorphous sets will be discussed.

References

[1] I. Glicksberg, Stone-Čech compactifications of products, *Trans. Amer. Math. Soc.* **90**: 369–382 (1959).

[2] P. Howard and J. E. Rubin, *Consequences of the axiom of choice*, Math. Surveys and Monographs **59**, A.M.S., Providence R.I., 1998.

[3] K. Keremedis and E. Wajch, Hausdorff compactifications in **ZF**, *https://arxiv.org/abs/1805.09708*: 1–32 (2018).

Davenport's constant for finite abelian groups with rank three

Maciej Zakarczemny

Cracow University of Technology, Kraków

e-mail: `mzakarczemny@pk.edu.pl`

Let G be a finite abelian group and $D(G)$ denote the Davenport constant of G. The precise value of the Davenport constant is known for p-groups and for groups of rank at most two, see [4], [5]. For general finite abelian groups the precise value is still unknown, see [3], [1].

We derive new upper bound for the Davenport constant for all groups of rank three. Our main result is that:

$$D(C_{n_1} \oplus C_{n_2} \oplus C_{n_3}) \leq (n_1 - 1) + (n_2 - 1) + (n_3 - 1) + 1 + (c_3 - 3)(n_1 - 1),$$

where $c_3 \leq 20369$ is a constant.

Therefore $D(C_{n_1} \oplus C_{n_2} \oplus C_{n_3})$ grows linearly with the variables n_1, n_2, n_3.

References

[1] A. Geroldinger, R. Schneider, *On Davenport's constant. Journal of Combinatorial Theory*, Series A, 61(1), (1992), 147-152.

[2] I. Rusa and A. Geroldinger, *Combinatorial Number Theory and Additive Group Theory* Advanced Courses in Mathematics CRM Barcelona, Birkh¨auser, Basel, 2009.

[3] F. Halter-Koch, *Factorization of algebraic integers*, Ber. Math.-Stat. Sektion Forschungszentrum Graz 191 (1983).

[4] J.E. Olson, *A combinatorial problem on finite abelian groups I*, J. Number Theory 1 (1969), 8-10.

[5] J.E. Olson, *A combinatorial problem on finite abelian groups II*, J. Number Theory 1 (1969), 195–199.