

Workshop
Modern
Applied
Mathematics

Kraków 2019


# orkshop odern pplied athematics 

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## Introduction

Workshop on Modern Applied Mathematics PK 2019 is the eighth edition of an annual conference on modern mathematics organized by the Department of Applied Mathematics of the Faculty of Computer Science and Telecommunications, Tadeusz Kościuszko Cracow University of Technology.

The Conference aims to present new results, to promote and to bring together researchers in the different research areas of mathematics and influence more cooperation among scientists working in mathematics. This conference will provide a unique forum for exchanging ideas and in-depth discussions on different aspects and different branches of mathematics.

You can find detailed information about conference on the site:

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www.wmam.pk.edu.pl
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We would like to thank professors Anna Denkowska, Maciej P. Denkowski, Aleksandra Nowel and Wiestaw Pawtucki for accepting invitation to give a lecture during the conference and all participants for interest in the eighth edition of our conference and scientific research in the field of mathematics and informatics

The conference is under the media patronage of Welcome Cracow.

We would like to express our thanks to the Vice-Rector for Research of Tadeusz Kościuszko Cracow University of Technology, prof. dr hab. inż. Tadeusz Tatara, the Dean of the Faculty of Computer Science and Telecommunications, dr inż. Jerzy R. Jaworowski, the Board of Directors and the Administration of the Department of Applied Mathematics as well as the staff of the Department of Applied Mathematics for their friendliness and support for conference organization.

On the behalf of the organizing committee
Grzegorz Gancarzewicz
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## Participants

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| Sebastian Baran | Tomasz Kowalczyk |
| Jakub Bielawski | Michat Kozdęba |
| Marcin Bilski | Robert Krawczyk |
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| Judyta Gojowy | Bartosz Sobolewski |
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| Monika Herzog | Nikola Zandecka |
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## Plenary Lectures

## Dynamic minimal spanning trees and their applications <br> Anna Denkowska anna.denkowska@uek.krakow.pl <br> Cracow University of Economics

This talk is part of the research on the interlinkages between insurers and their contribution to systemic risk in the insurance market. We will present the results of the analysis of linkage dynamics and systemic risk in the European insurance sector, which were obtained in [3] using correlation networks. These networks are constructed onthe basis of dynamic dependence structures modelled using a copula. Then, minimum spanning trees (MST) are determined. The linkage dynamics is described by means of selected topological network measures.

## References

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[3] S. Wanat and A. Denkowska, , Linkages and systemic risk in the European insurance sector: Some new evidence based on dynamic spanning trees, arXiv:1908.01142 (2019).

## Medial axes in the theory of singularities

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The medial axis $M_{X}$ of a nonempty closed set $X \subsetneq \mathbb{R}^{n}$ is the set of those points of $\mathbb{R}^{n}$ for which there is more than one point in $X$ realizing the Euclidean distance to $X$. It has been extensively studied since its introduction in the late sixities as the central concept of pattern recognition. However, its relation with the singularities of $X$ has been observed only recently in [3]. In this talk we will
be particularly interested in the characterisation of the set $X \cap \overline{M_{X}}$, i.e. the set of those singular points of $X$ which are reached by $M_{X}$, where $X$ is subanalytic or definable in an o-minimal structure.

## References

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[3] Maciej P. Denkowski, On the points realizing the distance to a definable set, J. Math. Anal. Appl. 378 no. 2: 592-602 (2011),
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## Efektywne metody liczenia niezmienników rzeczywistych odwzorowań wielomianowych

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Rozwój teorii baz Gröbnera w latach sześćdziesiątych XX wieku oraz wyniki Hironaki dotyczące rozwiązywania osobliwości przyczynity się do rozkwitu i wzrostu znaczenia metod efektywnego badania niezmienników związanych z obiektami badanymi w geometrii algebraicznej. Szybki rozwój komputerów umożliwit praktyczne użycie wypracowanych algorytmów.

Przedstawię zastosowanie klasycznych wyników dotyczących liczenia liczby pierwiastków wielomianów (formuła śladu - Pedersen, Roy, Szpirglas, Becker, Wörmann) oraz lokalnego stopnia topologicznego (Eisenbud, Levine, Khimshiashvili, Szafraniec, Łęcki) za pomocą sygnatury formy kwadratowej do obliczania pewnych niezmienników rzeczywistych odwzorowań wielomianowych.

## O twierdzeniu Coste'a-Reguiat

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Przedstawię dowód twierdzenia M. Coste'a i M. Reguiat z 1992 roku mówiącego, że dla dowolnej funkcji semialgebraicznej ciągłej $f: X \rightarrow \mathbb{R}$ określonej na zwartym semialgebraicznym podzbiorze $X$ przestrzeni $\mathbb{R}^{n}$ isnieje homeomorfizm semialgebraiczny $h: Y \rightarrow X$ określony na pewnym zwartym podzbiorze $Y$ przestrzeni $\mathbb{R}^{n}$ i taki, że zrówno $h$ jak i $f \circ h$ są odwzorowaniami Lipschitza.

## Roman domination in graphs and its variants

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Let $G=(V, E)$ be a simple graph. A set $D \subseteq V$ is a dominating set of $G$ if each vertex not in $D$ is adjacent to a vertex of $D$. The domination number of $G$ is the size of a smallest dominating set of $G$. A Roman dominating function on a graph $G=(V, E)$ is defined to be a function $f: V \rightarrow\{0,1,2\}$ satisfying the condition that every vertex $u$ for which $f(u)=0$ is adjacent to at least one vertex $v$ for which $f(v)=2$. The weight of a Roman dominating function $f$ is the value $f(V)=\sum_{u \in V} f(u)$. The minimum weight of a Roman dominating function on a graph $G$ is called the Roman domination number of $G$. We will study results on domination, Roman domination and their variants in some classes of graphs. This talk will also feature future possible ways of continuing the study of these parameters.

## References

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[3] M.A. Henning, S.T. Hedetniemi, Defending the Roman Empire - A new strategy, Discr. Math. 266 (2003) 239-251.

## Contributed Talks

## Optimizing the expected utility of dividend payments of an insurance company <br> Sebastian Baran sebastian.baran@uek.krakow.pl <br> Cracow University of Economics

The lecture will address the problem of maximizing the expected utility of dividend payments of an insurance company. F. Hubalek and W. Schachermayer in their paper [1] considered the above problem assuming that the risk process is modeled as a Brownian motion with drift. In this lecture, we will focus on the situation when the reserves of an insurance company are described by the classical Cramér-Lundberg model. The results of papers [2] and [3] will be presented.

## References

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[3] S. Baran, Z. Palmowski, Optimizing the expected utility of dividend payments for a Cramér-Lundberg risk process, Applicationes Mathematicae 44 (2017), 247-265.

## Evolutionary model of migrants' assimilation <br> Jakub Bielawski jakub.bielawski@uek.krakow.pl Cracow University of Economics

We present an attempt to combine two mechanisms - the assimilation of migrants and its impact on the well-being and choices of natives - in one model using the tools of game theory and dynamical systems. Namely, we formulate a co-evolutionary model: a system of differential equations describing the interaction between the choices of migrants and natives. In this model, both social groups interact through a relative deprivation channel that determines optimal
responses (strategies) for the current composition of the population (fraction of assimilating migrants and fraction of high-skilled natives) and their incomes.

We identify the evolutionary stable states, i.e. the states where the populations of migrants and natives are in equilibrium. In addition, we introduce a simple assimilation policy of the host country, financed from taxes imposed on the natives, which aims to increase the assimilation by reducing its cost for the migrants.

The main results of the analysis are as follows. First, we find that the decisions on assimilation of migrants and the skill formation of the natives are interrelated. Second, the group of migrants can become stuck in the non-effective equilibrium with no assimilation, if unaffected by assimilation policy. Third, we identify conditions under which an assimilation policy can bring the group of migrants to full assimilation and, moreover, increase the well-being of migrants and natives alike, compared to the no-assimilation outcome.

## Rational bases for monodromy groups of certain Calabi-Yau operators

Tymoteusz Chmiel tymoteusz.chmiel@gmail.com Jagiellonian University

With a family of Calabi-Yau threefolds $Y_{t}$ over a complement of a finite set $\Sigma \subset \mathbb{P}^{1}$ we may associate a period function $y(t):=\int_{\gamma_{t}} \omega_{t}$, where $\gamma_{t} \in H_{3}\left(X_{t}, \mathbb{Z}\right)$ and $\omega_{t} \in H^{3,0}\left(X_{t}, \mathbb{Z}\right)$. This function is annihilated by the Picard-Fuchs operator of the family, a differential operator of order four. One of the most important tools for studying such an operator is its monodromy group - the image of the representation of the fundamental group $\pi_{1}\left(\mathbb{P}^{1} \backslash \Sigma\right)$ given by continuation of a fundamental system of solutions along loops. For the simplest case of hypergeometric operators it is known that there is a basis of solutions (called the Doran-Morgan basis) such that the monodromy group is contained in the congruence subgroup of $\operatorname{Sp}(4, \mathbb{Z})$ and the transition matrix from this basis to the Frobenius basis at a point of maximal unipotent monodromy contains geometric invariants of the mirror of $Y_{t}$. I will present numerical computations of the monodromy groups of Picard-Fuchs operators of families of double octic Calabi-Yau threeefolds in the counterpart of the Doran-Morgan basis and of the transition matrices between this basis and Forbenius bases at certain singular points.

## References

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[2] C. Doran, J. Morgan, Mirror Symmetry and Integral Variations of Hodge Structure Underlying One Parameter Families of Calabi - Yau Threefolds, in: Mirror Symmetry V, 517 - 537, AMS/IP Stud. Adv. Math., 38, Amer.

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## Mechanisms leading to equilibrium in economy with financial market

Ilona Ćwieczek, Agnieszka Lipieta<br>Uniwersytet Ekonomiczny w Krakowie

This research extends the analysis of the economic activity in multi-market models of the market economy by introducing innovations in the production sector, taking into account the different ownership structure of firms (see [1]). Similarly to the construction of the income model with production (see [2]), the basic characteristics of the considered economic system and consequently its innovative extension are examined in two stages. Firstly, the impact of the real economy on innovation is analysed. Secondly, by adopting certain characteristics of the real sector as known, it is possible to analyse changes in the financial sector of the economy and their impact on the production activity of firms. As a result a set of some mechanisms which lead to equilibrium in the competitive economy with financial market and private ownership are modelled. The mechanisms under study are caused by innovative and non-innovative changes introduced within economic evolution.

## References

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## Why does the casino always win at roulette?

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The objective of the paper is to present experience with the roulette system, which should give a very good chance of winning. At the beginning the rules of roulette will be explained. In the following, the simulation written in the R environment will be discussed. Then the system according to which the simulation game will take place will be explained. Finally, conclusions, observations from the simulation and explanation of the problems of the given system will be given.

## References

[1] http://www.roulettestrategy.net/wheel/
[2] https://casino.betfair.com/game/european-roulette-cptn

## Is it possible to predict Bitcoin price?

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The aim of the presentation is comparing two (non typical) methods of time series prediction using the data set containing information about daily bitcoin prices from almost 6 years. A time series is a data set containing values pointed in equally spaced time. Bitcoin is a cryptocurrency, created by an unknown individual or group nicknamed Satoshi Nakamoto in virtual world. Two methods of prediction of bitcoin prices future values will be explained: random walks and Long Short-Term Memories (LSTM). The analysis of both methods turned out LSTM to give better prediction although it remains less well understood.

## References

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[2] Taniya Kabaye, An analysis of the Random Walk Hypothesis: Evidence from the Lusaka Stock Exchange
[3] Satoshi Nakamoto, Bitcoin: A Peer-to-Peer Electronic Cash System

## Some remarks on a family of semi-exponential operators

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We shall consider a family of semi-exponential type operators $W_{\lambda}$ defined as follows

$$
W_{\lambda}(f, t)=\int_{A}^{B} W(\lambda, t, u) f(u) d u
$$

where the kernel of $W_{\lambda}-W$ is a positive function satisfying the following partial differential equation

$$
\frac{\partial W}{\partial t}(\lambda, t, u)=\frac{\lambda}{p(t)} W(\lambda, t, u)(u-t)-\beta W(\lambda, t, u),
$$

and the normalization condition

$$
\int_{A}^{B} W(\lambda, t, u) d u=1
$$

## References

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[2] A. Tyliba and E. Wachnicki, On some class of exponential type operators, Comment. Math. (Prace Mat.) 45 (1): 59-73 (2005).

## Classification of generalized Calabi type Kähler surfaces

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In the talk we classify the important class of QCH Kähler surfaces- the generalized Calabi type Kähler surfaces. These are Kähler surfaces admitting an opposite Hermitian structure which is not conformally Kähler. In that way we present a large class of Hermitian surfaces with J-invariant Ricci tensor which are not conformally Kähler.

## References

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[6] W. Jelonek, E. Mulawa, Generalized Calabi type Kähler surfaces, arxiv (2019)

## Consumers' Optima in Schumpeterian Evolution

Marta Kornafel marta.kornafel@uek.krakow.pl
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We will consider the behaviour of consumers' optimal allocations in the result of Schumpeter evolution of economy. The goal of consumer in general equilibrium model is to choose optimal allocation that maximises his preferences over the budget set, given the price and initial allocation. The crucial point is to justify - taking into account the changing preferences - that the consumers choosing the optimal allocation at every stage of evolutionary process will end up at the optimal state of the final economy. In the talk we will provide the conditions,
under which the positive answer is possible.

Blown-up Čech cohomology and its applicatons

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In my talk I will define blown-up Čech cohomology and prove real algebraic version of Cartan's Theorem B for some quasi-coherent sheaves on a real affine algebraic variety. I will use this result to deduce universal solvability (after blowing-up) of the real algebraic First Cousin Problem.

## References

[1] T. Kowalczyk, Blown-up Čech Cohomology and Cartan's Theorem B on Real Algebraic Varieties, Int. J. Math. 30 (09) (2019), 1950042.

## Uniqueness of minimal projection in smooth three-dimensional matrix spaces

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Let $S=(M(n, m, r),$.$) be a space of functions f:\{1, \ldots, n\} \times\{1, \ldots, m\} \times$ $\{1, \ldots, r\} \mapsto \mathbb{K}$ with the norm .. We can consider it as a space of threedimensional matrices with real or complex elements. As $M(1,1, r)$ we understand the subspace of $S$ which consists of matrices whose elements $a_{i j k}$ satisfy $a_{i_{1} j_{k} k}=$ $a_{i_{2} j_{2} k}$ for every $i_{1}, i_{2} \in\{1,2, \ldots, n\}, j_{1}, j_{2} \in\{1,2, \ldots, m\}$ and $k \in\{1,2, \ldots, r\}$. We define $M(1, m, 1)$ and $M(n, 1,1)$ analogously. I will show that there is exactly one minimum projection of $S$ onto it's subspace $T=M(1,1, r)+M(1, m, 1)+$ $M(1,1, r)$. It is a generalization of the results of T. Skrzypek [1], [2] using my results [3].

## References

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[2] L. Skrzypek, Minimal projections in spaces of functions of $N$ variables, J. Approx. Theory, Vol. 123, (2003), 214-231.
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## On interesting subsequences of the sequence of primes

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It is a famous result that the set of quotients of prime numbers is dense in the set of positive real numbers. It is a motivation to wide study of denseness properties of subsets of positive integers on real half-line, see e.g. [1, 11, 12, 14]. One can meet it as an exercise on course of number theory, see [2, Problem 218], [3, Ex. 4.19], [9, Ex. 7, p. 107], [10, Thm. 4] and also in several articles, e.g. [4, Cor. 4], [6, Thm. 4], [13, Cor. 2] (according to the last reference, the result was known to Sierpiński, who credits it to Schinzel [8]). The authors of [4] generalized this result to the subsets of prime numbers in given arithmetic progressions.

Motivated by the article [5] on "light" subsets of positive integers (i.e. subsets with slowly growing counting fuctions) we focus on the family of subsets $\mathbb{P}_{k}=$ $\left\{p_{1}^{(k)}<p_{2}^{(k)}<p_{3}^{(k)}<\ldots\right\}, k \in \mathbb{N}$, of prime numbers such that every next set contains these elements of the preceding one indexed by prime numbers. As a consequence, every next set is a zero asymptotic density subset of the preceding one. Although the sets $\mathbb{P}_{k}$ are "lighter and lighter" as $k$ increases, we will show that all of them have dense quotient sets in the set of positive real numbers. We will also study the sets $\mathbb{P}_{n}^{T}=\left\{p_{n}^{(k)}: k \in \mathbb{N}\right\}, n \in \mathbb{N}$, and $\operatorname{Diag} \mathbb{P}=\left\{p_{k}^{(k)}: k \in \mathbb{N}\right\}$. We will prove that, in the opposition to the sets $\mathbb{P}_{k}$, their quotient sets are not dense in $\mathbb{R}_{+}$.

This is a joint work with János T. Tóth.

## References

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## Definable desingularization over Henselian fields with analytic structure and applications

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I deal with Henselian fields $K$ with analytic structure. Separated analytic structures (with tho kinds of variables), unlike the strictly convergent ones, admit reasonable quantifier elimination in the analytic languages. However, the rings of global analytic functions with two kinds of variables seem not to have good algebraic properties such as Noetherianity or excellence. Therefore the usual global resolution of singularities from rigid analytic geometry is no longer at our disposal. Two crucial tools of my approach are the closedness theorem from my papers $[2,3,4]$ and a definable version from [7] of Bierstone-Milman's canonical desingularization algorithm [1], carried out within a category of definable, strong analytic manifolds. Strong analytic objects are those definable ones that remain analytic over all fields elementarily equivalent to $K$. These tools will be applied to the existence of definable retractions or extending continuous definable functions (cf. [5,6,7]). The earlier techniques and approaches to the purely topological versions of those problems cannot be carried over to the definable settings because, among others, non-Archimedean geometry over non-locally compact fields suffers from lack of definable Skolem functions. All the results remain valid for strictly convergent analytic structures, whose classical examples are complete fields with the Tate algebras of strictly convergent power series.

## References

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## A new estimator of the Bayes factor in models with latent variables

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Formal Bayesian comparison of two competing models amounts to estimation of the Bayes factor equal to the ratio of marginal data density values (i.e. marginal likelihoods). Usually, in models with latent variables the marginal data density values are represented in the form of high-dimensional integrals over the parameter and latent variable space. The integrals are often computationally infeasible. Thus, other methods of evaluation of the Bayes factor, which do not require a direct calculation of the marginal data density values, are needed. In this paper a new method of estimation of the Bayes factor in models with a large number of latent variables is proposed. It is shown that the Bayes factor is equal to the posterior mean (restricted to a certain subset D of the parameter and latent variable space) of the ratio of conditional densities of the corresponding quantities times the reciprocal of the posterior probability of the subset D . This identity motivates the researcher to use arithmetic mean estimator of the ratio based on simulation from the posterior distribution, restricted to any (but reasonable) subset of the space of parameters and latent variables. By trimming this space, regions of relatively low variability of the ratio of densities under the posterior distribution are selected, and thereby the efficiency of the arithmetic mean estimator gets improved. Simulation examples illustrate that the proposed estimator performs very well. Next, the estimator is used to formally compare the hybrid IG-MSF-SBEKK model with the t-SBEKK specification.

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## Approximation theory and tame geometry <br> Rafat Pierzchała Rafal.Pierzchala@im.uj.edu.pl <br> Jagiellonian University

I will discuss how methods of tame geometry can be used to solve certain
problems in approximation theory and pluripotential theory.

## Some Grothendieck topologies in a simple language

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A smopology on a set $X$ is a family $\mathcal{L}_{X}$ of subsets of $X$ such that:
(S1) $\emptyset \in \mathcal{L} \chi$.
(S2) if $L, M \in \mathcal{L}_{X}$, then $L \cap M, L \cup M \in \mathcal{L}_{X}$,
(S3) $\forall x \in X \exists L_{x} \in \mathcal{L}_{X} x \in L_{X}$ (i. e., $\cup \mathcal{L}_{X}=X$ ).
Elements of $\mathcal{L}_{X}$ are called smops (small open sets). Each smopology determines
a concrete Grothendieck topology ( $G$-topology) which is a locally small DelfsKnebusch generalized topological space.

The main result is the following: The category of locally small generalized topological spaces and strictly continuous mappings is concretely isomorphic to the category of sets with smopologies and their bounded continuous mappings (see $[1,2]$ ).

Such spaces are used in o-minimal homotopy theory and in rigid analytic geometry.

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## On the last nonzero digits of $p$-adic analytic functions

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Recently, there has been some interest in the last nonzero digits of certain integer-valued sequences, such as $(n!)_{n \geq 0},\left(n^{n}\right)_{n \geq 0}$, and the Fibonacci numbers $\left(F_{n}\right)_{n \geq 0}$ (see for example [1, 3]). Motivated by these results, in this talk we consider in a similar context the sequence $(f(n))_{n \geq 0}$ of values of a $p$-adic locally analytic function $f: \mathbb{Z}_{p} \rightarrow \mathbb{Q}_{p}$, where $p$ is prime. In particular, we provide a necessary and sufficient condition for $p$-automaticity of the last nonzero digits in the $p$-adic expansion of $f(n)$, which turns out to be very similar to the condition for $p$-regularity of $p$-adic valuations $v_{p}(f(n))$ proved in [2]. We apply the results to show that the last nonzero digits of Fibonacci numbers in base $b$ (not necessarily prime) form a $b$-automatic sequence.

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## Semialgebraiczna wersja twierdzenia Calderóna-Zygmunda o regularyzacji funkcji odległości

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Jednym z użytecznych narzędzi w analizie jest twierdzenie Calderóna-Zygmunda o regularyzacji mówiące, że funkcja odległości od domkniętego podzbioru $W \subset$ $\mathbb{R}^{n}$ jest równoważna pewnej funkcji $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$, klasy $C^{\infty}$ na uzupetneniu zbioru $W$. Ponadto pochodne cząstkowe, do ustalonego rzędu, funkcji $f$ na zbiorze $\mathbb{R}^{n} \backslash W$ są kontrolowane przez funkcję odlegtości. Twierdzenie to ma szereg zastosowań m.in. w badaniu eliptycznych równań różniczkowych cząstkowych. Ze względu na rozwijające się zastosowania geometrii semialgebraicznej naturalnym i interesującym pytaniem jest czy twierdzenie to zachodzi w kategorii semialgebraicznej. Celem referatu jest przedstawienie pozytywnej odpowiedzi na to pytanie. Mianowicie przedstawimy następujące twierdzenie

Twierdzenie. Niech $W \subset \mathbb{R}^{n}$ będzie domkniętym semialgebraicznym podzbiorem, p liczbq naturalnq. Istniejq wtedy state $m, M, B_{\alpha} \in \mathbb{R}$ oraz funkcja $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$, która w restrykcji do zbioru $\mathbb{R}^{n} \backslash W$ jest klasy $\mathcal{C}^{\infty}$ oraz dla wszystkich $x \in \mathbb{R}^{n} \backslash W$ i każdego wielowskaźnika $\alpha \in \mathbb{N}^{n}$ o dtugości $|\alpha| \leq p$ zachodzq:

$$
\begin{align*}
& m \operatorname{dist}(x, W) \leq f(x) \leq M \operatorname{dist}(x, W),  \tag{1}\\
& \left|\frac{\partial^{|\alpha|}}{\partial x^{\alpha}} f(x)\right| \leq B_{\alpha}(\operatorname{dist}(x, W))^{1-|\alpha|} . \tag{2}
\end{align*}
$$

Na zakończenie pokaż emy kilka bezpośrednich zastosowań tego twierdzenia w geometrii semialgebraicznej.
Omawiane rezultaty są częścią wspólnej pracy z B. Kocel-Cynk i W. Pawtuckim.

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## On densely complete and sequential spaces in ZF

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The following new theorem, proved in [1] and [2], leads to solutions of two problems posed in [3] but resolved in [1] and [2]:

Theorem It holds in ZF that, for every positive integer n, the Euclidean space $\mathbb{R}^{n}$ is sequential if and only if $\mathbb{R}$ is sequential.

The main aim of this talk is to show distinct proofs to Theorem 1 and solutions of the above-mentioned problems from [3], as well as to comment on the independence of ZF of several sentences concerning densely complete metric spaces introduced in [1].

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## On the zero-sum constant, the Davenport constant and their analogues

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Let $D(G)$ be the Davenport constant of a finite Abelian group $G$. For a positive integer $m$ (the case $m=1$, is the classical one) let $E_{m}(G)$ (or $\eta_{m}(G)$, respectively) be the least positive integer $t$ such that every sequence of length $t$ in $G$ contains $m$ disjoint zero-sum sequences, each of length $|G|$ (or of length $\leq \exp (G)$ respectively). We prove that if $G$ is an Abelian group, then $E_{m}(G)=D(G)-1+m|G|$, which generalizes Gao's relation (see [1, Conjecture 6.5]). Moreover, we exam-
ine the asymptotic behavior of the sequences $\left(E_{m}(G)\right)_{m \geq 1}$ and $\left(\eta_{m}(G)\right)_{m \geq 1}$. We prove a generalization of Kemnitz's conjecture. At the and we make a natural conjecture in the non-Abelian case.

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## Primes with prime indices

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Let $n, k \in \mathbb{N}$ and let $p_{n}$ denote the $n$th prime number. We define $p_{n}^{(k)}$ recursively as $p_{n}^{(1)}:=p_{n}$ and $p_{n}^{(k)}=p_{p_{n}^{(k-1)}}$, that is, $p_{n}^{(k)}$ is the $p_{n}^{(k-1)}$ th prime.

The aim of the talk is to present results concerning the asymptotic behaviour of sequences Diag $\mathbb{P}:=\left(p_{k}^{(k)}\right)_{k=1}^{\infty}$ and $\mathbb{P}_{n}^{T}:=\left(p_{n}^{(k)}\right)_{k=1}^{\infty}$ for each fixed natural number $n$, and their counting functions. More precisely, we will show that

$$
\#(\operatorname{Diag} \mathbb{P} \cap[1, x]) \sim \#\left(\mathbb{P}_{n}^{T} \cap[1, x]\right) \sim \frac{\log x}{\log \log x}
$$

as $x \rightarrow \infty$.
In particular, we will give answers to some questions and prove a conjecture posed by Miska and Tóth in their recent paper [1].

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## Contents

Introduction ..... 5
Participants ..... 7
Plenary Lectures ..... 9
Anna Denkowska
Dynamic minimal spanning trees and their applications ..... 9
Maciej P. Denkowski
Medial axes in the theory of singularities ..... 9
Aleksandra Nowel
Efektywne metody liczenia niezmienników rzeczywistych odwzorowań wielomianowych ..... 10
Wiestaw Pawtucki
O twierdzeniu Coste'a-Reguiat ..... 10
Joanna Raczek
Roman domination in graphs and its variants ..... 11
Contributed Talks ..... 13Sebastian BaranOptimizing the expected utility of dividend payments ofan insurance company . . . . . . . . . . . . . . . . . . . . . . 13Jakub BielawskiEvolutionary model of migrants' assimilation13
Tymoteusz Chmiel
Rational bases for monodromy groups of certain Calabi- Yau operators ..... 14
Ilona Ćwieczek, Agnieszka Lipieta
Mechanisms leading to equilibrium in economy with fi- nancial market ..... 15
Andrzej Filek
Why does the casino always win at roulette? ..... 15
Kinga Głąbińska
Is it possible to predict Bitcoin price? ..... 16
Monika Herzog
Some remarks on a family of semi-exponential operators ..... 16
Włodzimierz Jelonek
Classification of generalized Calabi type Kähler surfaces ..... 17
Marta Kornafel
Consumers' Optima in Schumpeterian Evolution ..... 17
Tomasz Kowalczyk
Blown-up Čech cohomology and its applicatons ..... 18
Michat Kozdęba
Uniqueness of minimal projection in smooth three-dimensional matrix spaces ..... 18
Piotr Miska
On interesting subsequences of the sequence of primes ..... 19
Krzysztof Jan NowakDefinable desingularization over Henselian fields with an-alytic structure and applications20
Anna PajorA new estimator of the Bayes factor in models with latentvariables21
Rafat Pierzchata
Approximation theory and tame geometry ..... 21
Artur Piękosz
Some Grothendieck topologies in a simple language ..... 22
Bartosz Sobolewski
On the last nonzero digits of $p$-adic analytic functions ..... 22
Anna Valette
Semialgebraiczna wersja twierdzenia Calderóna-Zygmundao regularyzacji funkcji odległości23
Eliza Wajch
On densely complete and sequential spaces in ZF ..... 24
Maciej ZakarczemnyOn the zero-sum constant, the Davenport constant andtheir analogues24
Błażej Żmija
Primes with prime indices ..... 25

